Discrete and Computational Geometry, WS1415
Exercise Sheet "7": Chan's Technique and Dilations
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 2nd of December 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 18: Constants in Chan's method (4 Points)

We would like to find out how large the constants in the main lemma of Chan's randomized technique might become. We refer to the application of computing the dilation of a polygonal chain. For the dilation-of-the-chain computation we choose $\alpha = 1$ and $r = 4$ for a decomposition as suggested. If the decision algorithm takes $O(f(n))$ time, the randomized optimization algorithm takes $R \times f(n)$ expected time for a content R. Please analyze what R would be in the following way.

1. Assume that the decision algorithm runs in $R' \times n \log n$ time.
2. Choose an ϵ so that the precondition of Chan's technique will satisfied, e.g, $\frac{n \log n}{n^\epsilon}$ monotone increases in n and $(\ln r + 1)\alpha^\epsilon < 1$.
3. How many recursion steps l have to be done for your choice of ϵ?
4. Express constant R in terms of precise values of l, α and r and the variable parameter R.

Exercise 19: The Decomposition of a Polygonal Chain (4 Points)

Consider a polygonal chain C with n polygonal vertices, and let V be the set of polygonal vertices of C. For any two points $p, q \in C$, the dilation $\delta_C(p, q)$ between p and q in C is $\frac{|C_p^q|}{|\overline{pq}|}$, where C_p^q is the simple path between p and q in C, and the dilation δ_C of C is $\max_{p,q \in C} \delta_C(p, q)$. Let W be a subset of V, and let Q be a subchain of C. Furthermore, Let $\delta_C(W, Q)$ be $\min_{p \in W, q \in Q} \delta_C(p, q)$, and let $\delta_C^*(W, Q)$ be $\sup_{(p,q) \in W \times Q, \overline{pq} \cap Q = \emptyset} \delta_C(p, q)$.

- Please give an example in which there exists a pair of points, $p \in W$ and $q \in Q$ such that $\delta_C(p, q) = \delta_C(W, Q)$ but $\overline{pq}$ intersects C.
- please prove that if $\delta_C(W, Q) = \delta_C$, $\delta_C(W, Q) = \delta_C^*(W, Q)$.