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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 5

Problem 1

This task is meant to practice the application of Chernoff/Rubin bounds.

We are given n jobs that we need to execute, we have m machines to do so, and we assume that n is a multiple of m. Our scheduling consists of assigning n/m jobs to each machine, and the assignment is chosen uniformly at random.

The execution time of a job is random, too: With probability p, a job needs time 1, with probability $1-p$, it needs time k, and $k > 0$ is a some given constant. The execution time of a job is chosen independently of the execution times of all other jobs.

On each machine, the jobs are executed one after another. For machine $i \in \{1, \ldots, m\}$, let X_i be the time it takes until all jobs that are assigned to i are finished. Notice that $\mathbf{E}(X_i)$ is the same for all $i \in \{1, \ldots, m\}$, let $E = \mathbf{E}(X_1)$ be this value. The machines run in parallel. Let X be the random variable for the time that it takes until all machines stop working, i.e. $X = \max_{i=1,\ldots,m} X_i$.

Prove a statement of the following form by using a Chernoff/Rubin bound:

$$\mathbf{Pr}(|X - E| \geq t) \leq \frac{1}{n}.$$

Here, t is a term that you should choose appropriately. *Hint:* To enable the usage of the Chernoff/Rubin bound variant from the lecture, first define binomially distributed variables Y_i that are related to the X_i.

Problem 2

In the lecture, we saw that there is a permutation π for which the deterministic bit fixing strategy yields a routing with execution time $\Omega(2^n/n)$. Assume that instead of bit fixing paths as defined in the lecture, we use bit fixing paths where the bits are fixed in a random order. For each $i \in V_n$, the order is chosen uniformly at random from all permutations, and independently of the order chosen for all other $i' \in V_n$. Show that there is a permutation for which this routing has execution time $2^{\Omega(n)}$ with probability $1 - e^{-2^{\Omega(n)}}$. *Hint 1:* The same permutation as for the deterministic case works. *Hint 2:* You may use that Theorem 3.8 (2.) from the lecture still holds when $\mathbf{E}(X)$ is replaced by a lower bound $u \leq \mathbf{E}(X)$. *Hint 3:* You may use that it holds for all $i, j \in \mathbb{N}$, $i \geq j$, that

$$\left(\frac{i}{j}\right)^j \leq \binom{i}{j} \leq \left(\frac{i \cdot e}{j}\right)^j.$$