

Computer Science Lecture WS 2015/2016 Discrete and Computational Geometry

Chan's Randomized Technique and Geometric/Graph Dilations

1. What is a decision problem and what is an optimization problem? What is Chan's randomized technique? Explain the behind idea from the random-min algorithm, i.e., expected $O(Dr + E \log r)$ run time.
2. Give the precise definition of the notion of geometric/graph-theoretic dilation for a network!
3. What is the geometric interpretation of a point on a polygonal chain and what is the relation between the geometric dilation of a polygon chain and the lower envelope of transformed cones? How can we then apply additively weight Voronoi diagram to develop a decision algorithm?
4. How to use Chan's randomized technique and the decision algorithm for the geometric dilation of a polygonal chain to develop an optimization algorithm?

Abstract Voronoi Diagrams

1. Define abstract Voronoi diagrams, describe the motivation, and list several examples. What is an admissible bisecting curve system?
2. Let (S, \mathcal{J}) be a bisecting curve system. Please prove that the following assertions are equivalent.
 - If $p, q,$ and r are pairwise different sites in S , then $D(p, q) \cap D(q, r) \subseteq D(p, r)$ (Transitivity)
 - For each nonempty subset $S' \subseteq S$, $R^2 = \bigcup_{p \in S'} \overline{\text{VR}(p, S')}$
3. Define a conflict graph for the incremental construction of AVD, and prove that local test is enough, i.e., $e \cap \text{VR}(t, R \cup \{t\}) = e \cap \text{VR}(t, \{p, q, r\})$, where $R \subseteq S$, $t \in S \setminus R$, and e is the Voronoi edge between $\text{VR}(p, R)$ and $\text{VR}(q, R)$.

4. Describe how to compute $V(R \cup \{s\})$ from $V(R)$, i.e., how to insert a new site s .
5. Describe how to update the conflict graph, i.e., computing $G(R \cup \{s\})$ from $G(R)$.

Geometric Duality, k -sets, and k^{th} -order Voronoi diagrams

1. How do we count the number of 2-partitions of an n -point set which can be separated by a straight line?
2. How do we enumerate all $O(n^2)$ 2-partitions of an n -point set which can be separated by a straight line?
3. What is a k -set? How do we bound the total number of $\leq k$ -sets?
4. What are old and new Voronoi vertices of the k^{th} -order Voronoi diagram? What is the relation between the two kinds of Voronoi vertices? What are type-1 and type-2 Voronoi regions? How do the two kinds of Voronoi regions form from the previous-order Voronoi diagram?
5. Why can we derive a recursive formula for the complexity of the k^{th} -order Voronoi diagram? Please explain the reasons using old and new Voronoi vertices, old and new Voronoi edges, and type-1 and type-2 Voronoi regions.

Convexity and Lattice

1. What are affine subspace, affinely independent, affine combination, convex combination, convex hull? What is the relation between linear subspace and affine subspace?
2. What are Caratheodory's theorem, Radon's Lemma, and Helly's theorem? Please use Radon's lemma to prove Helly's theorem.
3. What are separation theorem for convex hulls and centerpoint theorem? Please prove the centerpoint theorem.
4. What is Minkowski's theorem? Please use an example to apply Minkowski theorem, e.g., forest visibility.

5. What is a general lattice and what is Minkowski's theorem for general lattices?
6. Please prove two-square theorem.

Convex Polytope and Arrangement

1. Please explain the central geometric duality in one dimension higher.
2. Please define V -polytope and H -polytope. Then please given an example whose description in V -polytope is much smaller than in H -polytope, and/or give the other way around.
3. What are faces of a convex polytope? Please also list some common names for specific dimensions
4. Please explain why the dual of a 3 dimensional convex polytope is a planar graph.
5. What is a simplicial polytope and what is a simple polytope? Please also explain their duality.
6. What is a cyclic polytope and what is its relation with the Gale's evenness criterion?
7. Sketch the upper bound theorem for the number of face of a convex polytope.
8. What is the complexity of an arrangement of n hyperplanes? Please sketch the analysis. (There are two methods, and you just need to give one).
9. What is the level of a point and what is the k -level of an arrangement?
10. Please sketch the analysis for the number of vertices of the k -level. (You can use the 2-dimensional case as an example.)
11. Please explain the zone theorem and give a rough idea for the proof (about the averaging argument)