Theoretical Aspects of Intruder Search

Course Wintersemester 2015/16
Escape Paths for the Intruder

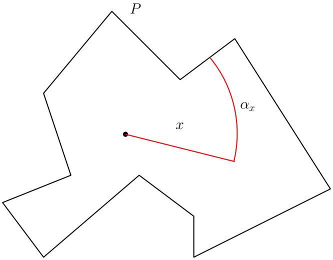
Elmar Langetepe

University of Bonn

February 2nd, 2016

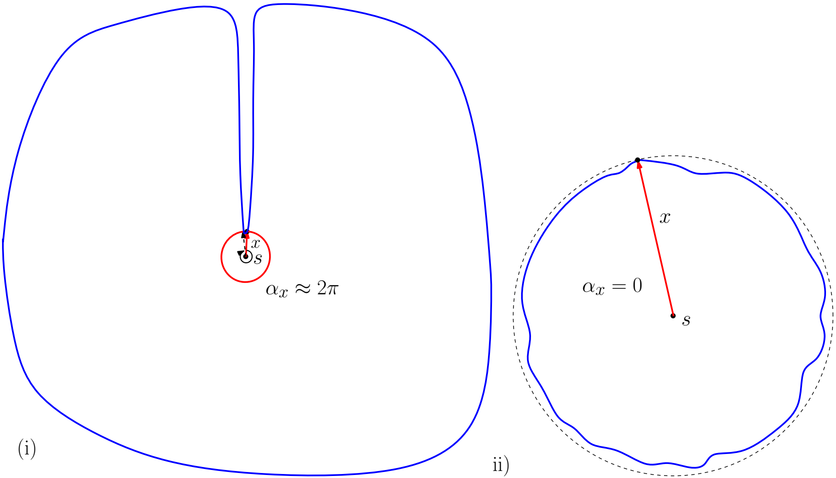
Different performance measure: Simple Polygon

- Simple polygon, escape path unknown
- Searching for different cost measure
- Polygonal extension of the list search problem
- Distance to the boundary x (estimation, given)
- Simple circular strategy $x(1 + \alpha_x)$



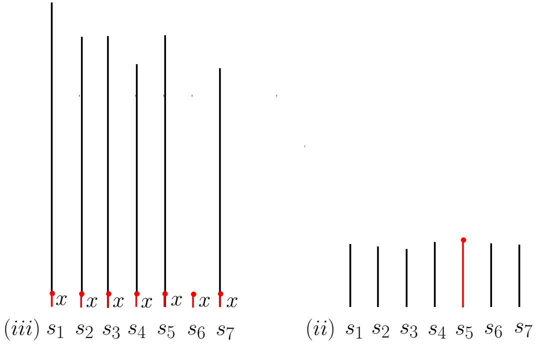
Extreme cases: Circular strategy

- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1 + 2\pi)$, $x_2(1 + 0)$



Discrete Version! Extreme Cases!

- Assume distribution is known!
- $f_1 \geq f_2 \geq \cdots \geq f_m$ order of the length given
- Extreme cases! $x_1(m)$, $x_2(1)$

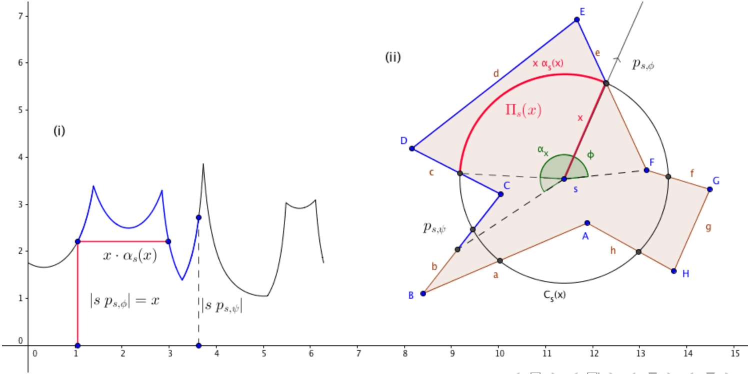


Circular strategy: Star shaped polygon

- Optimal circular espape path for $s \in P$: $\Pi_s(x)$
- For any distance x a worst-case $\alpha_s(x)$
- In total: $\min_x \;\; x(1+\alpha_s(x))$

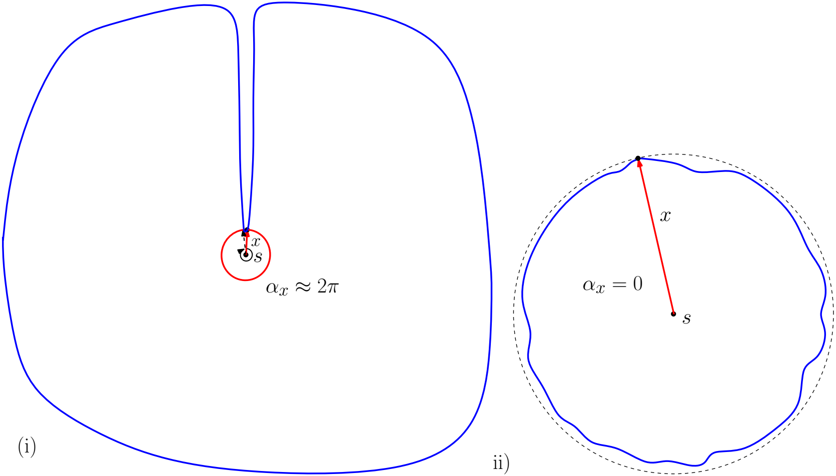
$$\Pi_s := \min_x \Pi_s(x) = \min_x x(1+\alpha_s(x)) \,.$$

- Radial dist. function interpretation: Area plus height!



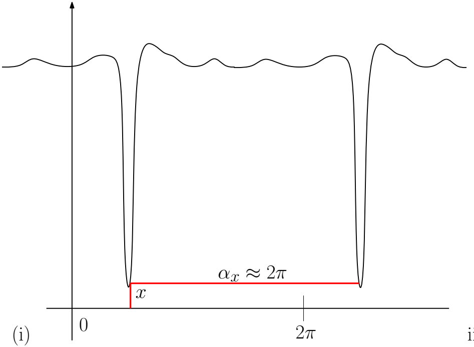
Extreme cases: Radial dist. function

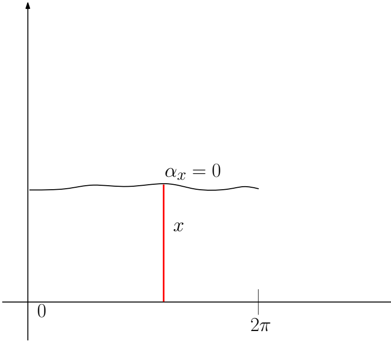
- Circular escape path: Distribution of the length is known
- Extreme situations: $x_1(1 + 2\pi)$, $x_2(1 + 0)$



Radial distance function of extreme cases

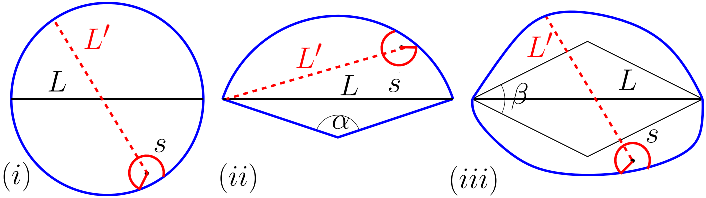
- Optimal circular espape path
- Hit the boundary by 90 degree wedge
- Area plus height! $\min_x \; x(1+\alpha_x)$





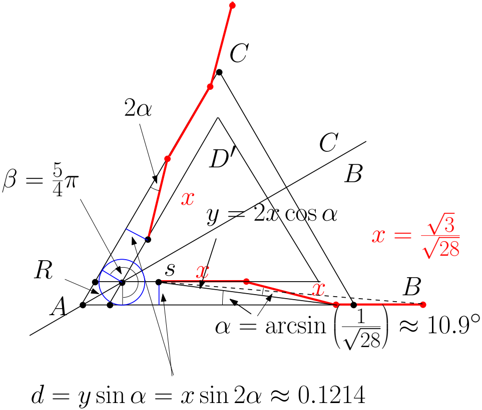
Different justifications

- Simple, computation (polynomial), star-shaped vs. convex
- Natural extension of the discrete certificate (Kirkpatrik)
- Outperforms escape paths for known cases (diameter)



Outperforms Zig-Zag path

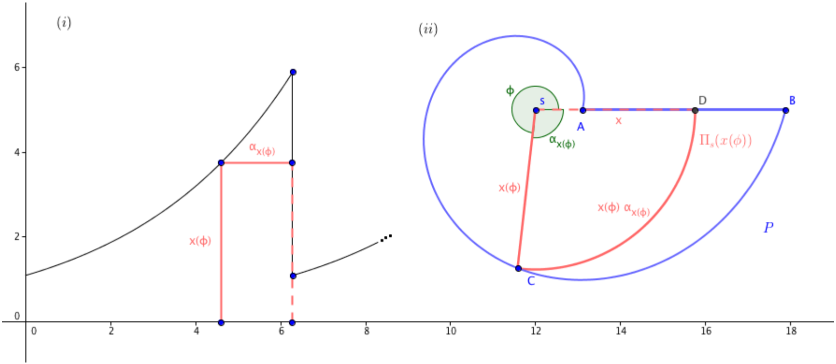
- For any position, better than the Zig-Zag path
- Formal arguments!
- Zig-Zag cannot end in farthest vertex: Region R!



$$0.125 \times (5\pi/4 + 1) < 2x = 2\frac{\sqrt{3}}{\sqrt{28}}$$

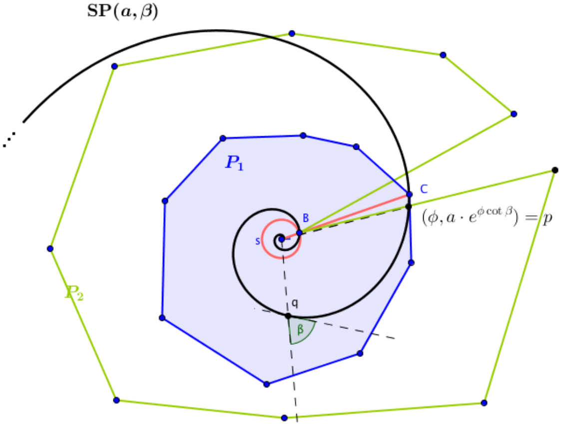
Interesting example

- Distance distribution exactly resembles the polygon
- Analogy to discrete case! Sorting!
- Log. spiral α_x for any x is known:
  $x(\phi) \cdot (1 + \alpha_{x(\phi)})$ with $\alpha_{x(\phi)} = 2\pi - \phi$ and $x(\phi) = A \cdot e^{\phi \cot \beta}$



Online Approximation!

- Inside a polygon P at point s, totally unknown
- Leave the polygon, compare to certificate path for $s \in P$
- Dovetailing strategy (discr. case)! Now spiral strategy (a, β)!



Analysis of a spiral strategy!

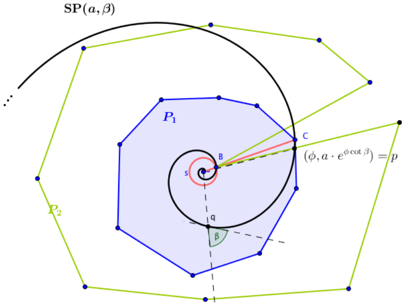
- Assume certificate: $x(1+\alpha_x)$ for s
- Spiral reach distance $x = a \cdot e^{(\phi-\alpha_x)\cot(\beta)}$ at angle ϕ
- Worst-case success at angle ϕ! (Increasing for α_x distances!)
- Ratio:

$$f(\gamma, a, \beta) = \frac{\frac{a}{\cos\beta} \cdot e^{\phi\cot\beta}}{a \cdot e^{(\phi-\gamma)\cot\beta}(1+\gamma)} = \frac{e^{\gamma\cot\beta}}{\cos\beta(1+\gamma)} \text{ for } \gamma \in [0, 2\pi]$$

- γ represents possible α_x!
- (β, a) represents the spiral strategy!
- Independent from a!
- How to choose β?

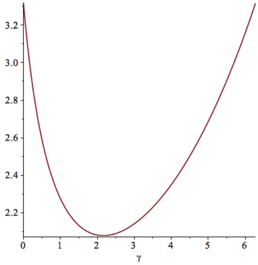
How to choose β?

- Ratio: $f(\gamma,\beta) = \frac{e^{\gamma \cot \beta}}{\cos\beta(1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- Balance: Choose β s.th. extreme cases have the same ratio
- $f(0,\beta) = \frac{1}{\cos\beta} = \frac{e^{2\pi \cot\beta}}{\cos\beta(1+2\pi)} = f(2\pi,\beta)$
- $\beta = \text{arccot}\left(\frac{\ln(2\pi+1)}{2\pi}\right) = 1.264714\ldots$



Balance the extreme cases!

- $\beta := \operatorname{arccot}\left(\frac{\ln(2\pi+1)}{2\pi}\right) = 1.264714\ldots$
- Ratio: $f(\gamma,\beta) = \frac{e^{\gamma \cot \beta}}{\cos\beta(1+\gamma)}$ for $\gamma \in [0, 2\pi]$
- $f(0,\beta) = f(2\pi,\beta) = 3.31864\ldots$
  and $f(\gamma,\beta) < 3.31864\ldots$ for $\gamma \in (0, 2\pi)$



Spiral strategy for $\beta = 1.264714\ldots$

Theorem 76: There is a spiral strategy for any unknown starting point s in any unknown environment P that approximates the certificate for s and P within a ratio of 3.31864.