Problem Set 7

Problem 1

Show that the recurrence

\[ h_0 = 1 + h_1, \quad h_n = 0, \quad \forall j \in \{0, \ldots, n-1\} : h_j \leq 1 + \frac{2}{3} h_{j-1} + \frac{1}{3} h_{j+1} \]

implies that \( h_j \leq 2^{n+2} - 2^{j+2} - 3(n-j) \) for all \( j \in \{0, \ldots, n\} \). Hint: First show by induction that for all \( j \in \{0, \ldots, n-1\} \), \( h_j \leq h_{j+1} + 2^{j+2} - 3 \) holds.

(This task completes the proof of Lemma 4.7).

Problem 2

Let \( a \in \{0,1\}^n \) be uniformly chosen from all possible assignments \( \{0,1\}^n \), and let \( a^* \) be an unknown but fixed assignment. Let \( r := |\{i \in \{1, \ldots, n\} \mid a_i = a^*_i\}| \) be the number of positions where \( a \) and \( a^* \) agree. Show that

\[ \Pr(r \geq n/2) \geq 1/2. \]