Problem Set 11

Problem 1

Consider an arbitrary binary optimization problem with linear objective $c^T x$ and solution set $S \subseteq \{0,1\}^n$ as discussed in Chapter 7. Recall that the winner gap $\Delta$ is defined as

$$\Delta := c x^* - c x^{**}$$

where $x^*$ is an arbitrary optimal solution and $x^{**}$ is a solution that is optimal amongst all solutions in $\{x \in S \mid x \neq x^*\}$. Find better upper bounds on $\Pr(\Delta \leq \epsilon)$ than the bound provided by Lemma 7.3 for the following scenarios:

1. The $c_i$ are $\phi$-perturbed numbers from $[0,1]$ (instead of $[-1,1]$).
   Show that $\Pr(\Delta \leq \epsilon) \leq n\phi\epsilon$.

2. The $c_i$ are numbers from $[1,e]$ that are chosen independently from the distribution with the density
   $$f(x) = \begin{cases} \frac{1}{x} & \text{for all } x \in [1,e] \\ 0 & \text{else.} \end{cases}$$
   Show that $\Pr(\Delta \leq \epsilon) \leq n \ln(1 + 2\epsilon)$.

Problem 2

Discuss how the SSP algorithm performs on the following input network. For each edge, the first number is the capacity, the second number is the cost. The value $T$ is a some integer.