Problem Set 3

Please hand in your until Monday April 30th.

Problem 1

1. A lively monkey types $26^6 \cdot 42 + 5$ letters (=499017796 letters) on a keyboard. We assume that the keyboard has only upper-case letters and that each of the 26 letters is chosen uniformly at random. What is the expected number of times that the word RANDOM appears?

2. We flip a fair coin $n + \log_2 n$ times, assume that $n$ is a power of two. We get a sequence $x_1, x_2, \ldots, x_{n + \log_2 n}$ with $x_i \in \{H, T\}$. We say that $x_i, x_{i+1}, \ldots, x_{i+\ell-1}$ is an $\ell$-sequence if $x_i = x_{i+1} = \ldots = x_{i+\ell-1}$ (all heads or all tails). What is the expected number of $\ell$-sequences for $\ell = 1 + \log_2 n$?

Problem 2

As in the proof of Theorem 1.14 (success probability of Karger’s Contract algorithm) let $A_i$ be the event that the algorithm contracts a good edge in iteration $i$. Show or disprove that $A_i$ and $A_j$ are generally independent for $i, j \in \{1, \ldots, n-1\}$.

Problem 3

In this task, we want to cut a graph $G = (V, E)$ into $r$ pieces instead of cutting it into two pieces as in the lecture. The parameter $r \in \mathbb{N}$ is a constant. We say that $r$ disjoint subsets $V_1, \ldots, V_r$ with $V = \bigcup_{i=1}^r V_i$ are an $r$-cut of $G$. We pay for all edges between these subsets, our cost is: $\frac{1}{2}(|\delta(V_1)| + |\delta(V_2)| + \ldots + |\delta(V_r)|)$. We want to find an $r$-cut with minimum cost. Generalize Karger’s Contract algorithm such that it finds a minimum $r$-cut with probability $\Theta(1/n^3r)$.

Problem 4

What is the running time of the FastCut algorithm (without repetitions) when we set $t := (3/4)n$? You may use the ‘master theorem’ (this theorem is explained in many books and lecture notes, see for example the notes from Avrim/Manuel Blum’s course at https://www.cs.cmu.edu/~avrim/451f11/lectures/lect0901.pdf).
**Master Theorem**

The recurrence

\[
T(n) = aT(n/b) + cn^k
\]

\[
T(1) = c,
\]

where \(a, b, c\) and \(k\) are all constants, solves to:

\[
T(n) \in \Theta(n^k) \text{ if } a < b^k
\]

\[
T(n) \in \Theta(n^k \log n) \text{ if } a = b^k
\]

\[
T(n) \in \Theta(n^{\log_a b}) \text{ if } a > b^k.
\]