Complexity of Boolean functions
SS 2018
Homework 4
14.05.2015

Exercise 1:
Prove that each prime implicant of a monotone function $f \in M_n$ contains only nonnegated variables.

Exercise 2:
Prove Theorem 3.1 of the lecture.

Exercise 3:
Let $g = \text{res}_\beta(v)$ for a gate $v$ in a monotone network $\beta$ for the function $f$. Prove the following assertions:

a) $g$ can be replaced by the constant zero iff for all $t \in PIM(g)$ for all monomials $t'$ there holds $tt' \notin PIM(f)$.

b) $g$ can be replaced by the constant one iff for all functions $h$ there hold $gh \leq f$ implies $h \leq f$. 