Exercise 1:

a) Show that Claim 1 and Claim 2 at page 161 of the lecture notes imply the lower bound \( \frac{1}{2} (\frac{m}{s-r})^{s/2} \).

b) Prove Corollary 4.2 of the lecture.

Exercise 2:

Develop an algorithm which computes for each node \( g \) of a given monotone network \( \beta \) \( \text{DNF}_\beta(g) \) and \( \text{CNF}_\beta(g) \).

Exercise 3:

a) Describe a CNF/DNF-switch.

b) Let \( \alpha \) be a DNF-formula (CNF-formula). Prove that the formula \( \gamma \) obtained by a DNF/CNF-switch (CNF/DNF-switch) computes the same function as \( \alpha \).

Exercise 4:

Consider the lower bound proof for the clique function which uses DNF/CNF-approximators.

a) Show that the number of inputs in \( T_1 \) for which the approximator \( D_g^r \) could introduce an error is bounded by \( \binom{m-r}{s-r} \left( \frac{m}{4s} \right)^r \).

b) Show that the number of inputs in \( T_0 \) for which the approximator \( C_g^k \) could introduce an error is bounded by \( (\frac{m}{2})^k (s-1)^{m-k} \).

c) Show that either \( C_{g_0}^k \) computes the constant function one or \( C_{g_0}^k \) computes the value of at least half of the inputs in \( T_1 \) incorrectly.

d) Prove Theorem 5.1 of the lecture.