Algorithmic Game Theory and the Internet
Summer Term 2019
Exercise Set 3

**Exercise 1:**

Consider the local search problem *Positive Not-All-Equal k-Sat* (Pos-NAE-kSAT) which is defined the following way:

**Instances:** Propositional logic formula with \( n \) binary variables \( x_1, \ldots, x_n \) that is described by \( m \) clauses \( c_1, \ldots, c_m \). Each clause \( c_i \) has a weight \( w_i \) and consists of exactly \( k \) literals, which are all positive (i.e., the formula does not contain any negated variable \( \overline{x}_i \)).

**Feasible solutions:** Any variable assignment \( s \in \{0, 1\}^n \)

**Objective function:** Sum of weights of clauses \( c_i \) in which not all literals are mapped to the same value.

**Neighbourhood:** Assignments \( s \) and \( s' \) are *neighbouring* if they differ in the assignment of a single variable.

(a) Show: Pos-NAE-kSAT is in PLS.

(b) Show: Pos-NAE-2SAT \( \leq_{PLS} \) MaxCut

(b) Show: Pos-NAE-3SAT \( \leq_{PLS} \) Pos-NAE-2SAT

**Exercise 2:**

We define a Congestion Game to be *symmetric*, if \( \Sigma_1 = \ldots = \Sigma_n \). Let \( PNE_{\text{Cong. Game}} \) and \( PNE_{\text{Sym. Cong. Game}} \) be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: \( PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}} \).

**Hint:** Add an auxiliary resource for each player with a suitable delay function.

Exercises 3 and 4 on the next page.
Exercise 3:  
(2+2 Points)
Consider the following cost-minimization game. Two car drivers approach a junction. Both drivers can either stop at (S) or cross (C) the junction. If a driver decides to stop, small costs emerge to her because of the waiting time. If both drivers decide to cross the junction, they will crash – resulting in high costs for both drivers.

\[
\begin{array}{c|cc}
 & C(\text{ross}) & S(\text{top}) \\
\hline
C(\text{ross}) & 100 & 1 \\
S(\text{top}) & 0 & 1
\end{array}
\]

(a) List all pure and mixed Nash equilibria.

(b) State a \textit{coarse-correlated equilibrium} that is not a pure or mixed Nash equilibrium.

\textbf{Hint:} Think of a probability distribution \( p \) “implementing” traffic lights.

Exercise 4:  
(2 Points)
Let \( p, p' \) be coarse correlated equilibria of a cost-minimization game \( \Gamma \). Prove that any convex combination of the distributions \( p \) and \( p' \) yields also coarse correlated equilibrium of \( \Gamma \) (i.e., any distribution \( q := \lambda p + (1 - \lambda)p' \) for a \( \lambda \in [0, 1] \)).