Exercise 1:
A vertex cover $X$ of a graph $G = (V, E)$ contains for every edge $e \in E$ at least one of its end nodes. A vertex cover $X \subseteq V$ is minimum if $|X| \leq |X'|$ for all vertex covers $X' \subseteq V$. Show that in bipartite graphs the size of a maximum matching is equal to the size of a minimum vertex cover.

Exercise 2:

a) Work out the algorithm for the computation of a maximum weighted matching in bipartite graphs in detail.

b) Show that the algorithm $\text{MaxWeiMatching}$ can be implemented such that its run time is $O(n^3)$ where $n = |A \cup B|$.

Exercise 3:
Construct a weighted undirected graph with the property that a matching of maximum weight is not a matching of maximum size.

Exercise 4:
Let $M_1$ and $M_2$ be two distinct matchings of a graph $G$ such that $|M_1| = r$, $|M_2| = s$ and $r \leq s$. Show that there exist two distinct matchings $M'_1$ and $M'_2$ such that
\[
\left\lfloor \frac{s + r}{2} \right\rfloor \leq |M'_1| \leq |M'_2| \leq \left\lceil \frac{s + r}{2} \right\rceil.
\]