Exercise 1:

Develop an algorithm for the solution of linear programs

\[ \min z(x) = c^T x \]
\[ Ax = b \]
\[ x \geq 0 \]

with \( A \) is an \((m \times n)\)-matrix and \( \text{rank}(A) < m \).

Exercise 2:

Consider the following linear program LP:

\[ \min z(x) = x_1 + x_3 \]
\[ x_1 + 2x_2 \leq 5 \]
\[ x_2 + 2x_3 = 6 \]
\[ 2x_1 - 3x_2 \geq 3 \]
\[ x_1, x_2, x_3 \geq 0 \]

a) Solve LP using the simplex algorithm.

b) Determine the dual linear program LP' of LP.

c) Present the complementary slackness condition and use this condition to get a solution of LP'. Evaluate the optimal costs of LP and LP' to examine your solution.

Exercise 3:

Replace in LP the inequality by \( x_1 + 2x_2 \leq -5 \) and repeat Exercise 2 for the resulting linear program.