Exercise 1:
Let us consider depth-first search of a directed graph. Let \( Q \) be the stack which is used for the organization of the search. Show that during the depth-first search the stack \( Q \) always contains a simple path from the start node to the top node of \( Q \).

Exercise 2:
   a) Develop an algorithm which decides in linear time if a given graph \( G = (V, E) \) is bipartite or not.
   b) Develop an algorithm which decides if a given graph \( G = (V, E) \) is bipartite or not after the deletion of one edge. What is the run time of your algorithm? Can you give a linear time algorithm for this problem?

Exercise 3:
Construct a graph which contains a maximum matching of size ten and a maximal matching of size five.

Exercise 4:
Let \( G = (V, E) \) be an undirected graph, \( M \) a maximal and \( M' \) a maximum matching of \( G \). Prove \( |M| \geq \frac{|M'|}{2} \).