Exercise 1: (3 Points)
Prove Observation 20.4: If $R$ is $\sigma$-strongly convex and $f_1, f_2, \ldots$ are convex then $R + \sum_t f_t$ is $\sigma$-strongly convex.

Exercise 2: (4 Points)
We consider Online Linear Regression as introduced in the lecture on December 18. Recall that

$$f_t(w_1, w_2) = \left(w_1 x^{(t)} + w_2 - y^{(t)}\right)^2.$$

Derive a regret bound for Follow-the-Regularized-Leader with Euclidean regularization under the assumption that $|x^{(t)}|, |y^{(t)}| \leq 1$ for all $t$ and $S = \{w \in \mathbb{R}^2 \mid \|w\|_2 \leq r\}$.

Exercise 3: (3 Points)
Derive a regret bound for Follow-the-Regularized-Leader if the Lipschitz constant depends on the time step, that is,

$$f_t(u) - f_t(v) \leq L_t \|u - v\| \quad \text{for all } u, v \in S.$$

Exercise 4: (3 Points)
Consider a finite set $X$. Show that in this case every hypothesis class $\mathcal{H}$ is PAC learnable. Use results from the lecture on January 8 but not from one on January 10.

Exercise 5: (4+3 Points)
Let $X = \mathbb{R}$ and let $\mathcal{H}$ be the hypotheses of the for $h(x) = 1$ for $x \in [a, b]$, $h(x) = 0$ otherwise. Give the growth function $\mathcal{H}[m]$ and prove your claim as follows.

(a) Show that for every set $S$ with $|S| = m$ it holds that $\mathcal{H}[S] \leq \mathcal{H}[m]$.

(b) For every $m$, give a set $S$ with $|S| = m$ such that $\mathcal{H}[S] = \mathcal{H}[m]$. Justify your claim.