Exercise 1:
Let $G = (V, E)$ be a directed graph. What could be the change of the number of strongly connected components of $G$ if we delete (insert) an edge?

Exercise 2:
Let $G = (V, E)$ be a directed graph. Let $G_{\text{red}} = (V_{\text{red}}, E_{\text{red}})$ be the reduced graph of $G$ defined as in the lecture. Develop a linear time algorithm for the computation of the reduced graph for a given directed graph $G$. Take care that $E_{\text{red}}$ does not contain multiple edges. Prove the correctness of the algorithm.

Exercise 3:
The transitive closure of a directed graph $G = (V, E)$ is a directed graph $H = (V, E')$ such that for $v, w \in V$, the edge $(v, w) \in E'$ iff there is a path from $v$ to $w$ in $G$. Develop an algorithm for the computation of the transitive closure of a given directed graph. What is the time used by your algorithm?

Exercise 4:
A directed graph $G = (V, E)$ is called half connected if for every $u, v \in V$ always a path from $u$ to $v$ or a path from $v$ to $u$ exists. Design an efficient algorithm which decides if a given graph $G$ is half connected. Prove the correctness of your algorithm and analyze its time complexity.