Exercise 1:
Prove Theorem 3.2 of the lecture.

Exercise 2:

a) Prove the following statement: If two different bases correspond to the same feasible basic solution \( x \) then \( x \) is degenerate.

b) Prove that there exists degenerate feasible basic solutions with unique corresponding basis.

Exercise 3:
Solve the following linear program using the simplex algorithm. \(<>\) means unconstrained.

\[
\begin{align*}
\text{max } z(x) &= x_1 - 3x_2 + x_3 \\
3x_1 + 2x_2 &= 6 \\
4x_1 + x_2 + 4x_3 &= 12 \\
x_1 &< 0 \\
x_2, x_3 &\leq 0
\end{align*}
\]

Exercise 4:
Solve the following linear program using the simplex algorithm.

\[
\begin{align*}
\text{min } z(x) &= 6x_1 - 9x_2 \\
x_1 - x_2 &= 6 \\
3x_1 + x_2 &\geq 1 \\
2x_1 - 3x_2 &\geq 3 \\
x_1 &\geq 0 \\
x_2 &\leq 0
\end{align*}
\]