1 **Affine Hull**

- For any set $X \subseteq \mathbb{R}^d$, let $\text{aff}_n(X)$ denote the intersection of all affine subspaces of $\mathbb{R}^d$ containing $X$.

- For any set $X \subseteq \mathbb{R}^d$, let $\text{aff}_c(X)$ denote the set of all affine combinations of points of $X$.

Prove that $\text{aff}_n(X) = \text{aff}_c(X)$.

2 **Convex Hull**

i) Prove that the set of all convex combinations of $x_1, \ldots, x_n \in \mathbb{R}^d$ is a convex set.

ii) Prove by induction on $n$ that the set of all convex combinations of $x_1, \ldots, x_n \in \mathbb{R}^d$ is contained in the convex hull of the set $\{x_1, \ldots, x_n\}$.

iii) Prove that for any closed set $X$ the convex hull $\text{conv}(X)$ is equal to the intersection of all closed halfspaces that contain $X$.

3 **Translated copies**

Let $K \subseteq \mathbb{R}^d$ be a convex set and let $C_1, \ldots, C_n \subseteq \mathbb{R}^d$, $n \geq d+1$, be convex sets such that the intersection of every $d+1$ of them contains a translated copy of $K$. Prove that the intersection of all sets $C_i$ also contains a translated copy of $K$ that is $\exists t \in \mathbb{R}^d: \{t + x \mid x \in K\} \subseteq \bigcap_{i=1}^n C_i$. 