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Randomized Algorithms and
Probabilistic Analysis
Summer 2017

Problem Set 2

Please hand in your solutions in Monday's lecture on the 8th of May (or via e-mail).

Problem 1

Alice tells Bob a new game. She has three six-sided dice. The dice are fair (all sides come up with the same probability), but they do not have the standard numbering. Instead, they have the following numbers:

- die A: 1,1,6,6,8,8
- die B: 2,2,4,4,9,9
- die C: 3,3,5,5,7,7

Alice explains that she lets Bob pick a die first to give him an advantage. Then she will pick a die. Then they roll their dice, and the player with the higher number wins. After playing a while, Bob thinks he is unlucky because Alice wins more than him. Is Bob unlucky or does Alice have a higher win chance? Alice uses the following strategy:

- If Bob picks A, she picks B.
- If Bob picks B, she picks C.
- If Bob picks C, she picks A.

Problem 2

Let `ALG` be a randomized algorithm with running time $\Theta(n^3 + n + \sqrt{n})$ that outputs an optimal solution (for an unspecified optimization problem) with probability at least $\frac{1}{\sqrt{n}\log n}$. Give a number ℓ of independent repetitions such that repeating `ALG` ℓ times and returning the best solution results in an algorithm with success probability at least $1 - \frac{1}{n^7}$. What is the running time of the resulting algorithm?

Problem 3

What is the running time of the `FastCut` algorithm (without repetitions) when we set $t := (3/4)n$? You may use the 'master theorem' (this theorem is explained in many books and lecture notes, see for example the notes from Avrim/Manuel Blum's course at `https://www.cs.cmu.edu/~avrim/451f11/lectures/lect0901.pdf`).

Problem 4

In this task, we want to cut a graph $G = (V, E)$ into r pieces instead of cutting it into two pieces as in the lecture. We say that r disjoint subsets $V_1, \ldots, V_r$ with $V = \cup_{i=1}^{r} V_i$ are an *r-cut* of G. We pay for all edges between these subsets, our cost is: $\frac{1}{2}(|\delta(V_1)|+|\delta(V_2)|+\ldots+|\delta(V_r)|)$. We want to find an r-cut with minimum cost.

Generalize Karger's `Contract` algorithm such that it finds a minimum r-cut with probability $\Theta(1/n^{3r})$.