Institut für Informatik
Prof. Dr. Heiko Röglin
Dr. Melanie Schmidt

universitätbonn

Randomized Algorithms and
Probabilistic Analysis
Summer 2017

Problem Set 4

Please hand in your solutions in Monday's lecture on the 22nd of May (or via e-mail).

Problem 1

Can Markov's inequality be improved? Show the following statement. For any $a > 1$, it is possible to define a random variable X_a that is non-negative, $\mathbf{E}(X_a)$ exists, and it holds that

$$\mathbf{Pr}(X_a \geq a \cdot \mathbf{E}(X_a)) = 1/a.$$

Problem 2

Which of the following algorithms compute a 2-approximation for the vertex cover problem? For each algorithm, give a counter example or argue why it provides a 2-approximation.

- Start with the empty cover. As long as there are uncovered edges, pick an uncovered edge. Add one arbitrary end point of the edge to the cover.
- Start with the empty cover. As long as there are uncovered edges, pick an uncovered edge. Add *both* end points of the edge to the cover.
- Compute a maximal matching $M \subseteq E$. For every $e \in M$, add both end points to the cover. [A matching is a set of edges that contains no adjacent edges].
- Compute a maximum cut $S \subset V$. Use S as the cover.
- Run a depth first search and let T be the DFS tree of the search. Let I be the set of internal nodes of T, use I as the cover.