Institut für Informatik
Prof. Dr. Heiko Röglin
Dr. Melanie Schmidt

UNIVERSITÄT BONN

Randomized Algorithms and
Probabilistic Analysis
Summer 2018

Problem Set 4

Please hand in your solutions until Monday May 7th.

Problem 1

At the end of a semester one of your other lectures has an exam which consists of 10 multiple choice questions (We have an oral exam). Each question has 4 possible answers, out of which exactly 1 is correct. In order to pass the exam you need to answer at least 5 questions correctly.

- Since you have studied for the exam, the chance that you know the answer to a question is 80% (This chance is independent for all 10 questions). If you know the answer to a question, you will be correct with probability 1. For all questions to which you do not know the answer you guess an answer uniformly at random. What is the expected number of questions you will answer correctly and what is the probability that you pass the exam?
- You know that one of your fellow students did not study at all for the exam and will guess on every question. What is the probability that your fellow student passes the exam?
- How do the probabilities change, when the correct solution to a question is not exactly one of the answer, but a subset of the four possible answers and all guesses are uniformly at random from the set of all subsets?

Problem 2

We are given a data stream of numbers $a_1, a_2, a_3, \ldots$ (of unknown length) and want to sample one number s. Instead of ReservoirSampling, we use the following algorithm: Intially, we store a_1 in s. Then, after each a_i, we keep the current s with probability $1/2$ and replace it with a_i with the remaining probability. What is the probability $\Pr(s = a_j)$ for $j \in \{1, \ldots, j\}$ after processing a_i?

Problem 3

Give an example of two random variables $X, Y : \Omega \to \mathbb{N}$ where $E(X), E(Y)$ and $E(X \cdot Y)$ exist, but $E(X \cdot Y) \neq E(X) \cdot E(Y)$. Bonus task: Give an easy example of two random variables $X, Y : \Omega \to \mathbb{R}$ where $E(X)$ and $E(Y)$ exist, but $E(X \cdot Y)$ does not exist.

Problem 4

We consider the third part of Example 2.2 from the lecture (page 29 in the lecture notes). Compute the expected value of Alice's gain in Example 2.2, i.e. compute $E(Y)$ for $Y : \Omega \to \mathbb{N}$ with

$$Y(H^{i-1}T) = \begin{cases} i & \text{if } i \text{ is odd} \\ -i & \text{if } i \text{ is even.} \end{cases}$$

On expectation, does Alice lose or gain points?