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Randomized Algorithms and
Probabilistic Analysis
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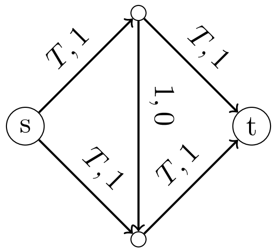
Problem Set 11

Problem 1

Discuss how the SSP algorithm performs on the following input network. For each edge, the first number is the capacity, the second number is the cost. The value T is a some integer.



Problem 2

Recall the instance from problem 1.

1. Give a short proof that changing the costs of the five edges by choosing them according to independent density functions $f_e : [0,1] \to [0,\phi]$ implies that the SSP algorithm converges in a constant number of steps for any integer T. Assume that Property 8.9 from the lecture is proven and use it.
2. Give an even shorter proof that uses Property 8.9 and Corollary 8.3.
3. Extend your proof from 2. to arbitrary input graphs of constant size.

Problem 3

Let $G = (V, E)$ be a graph let $m = |E|$ be the number of edges. Let $\mathcal{S}$ be the family of all feasible matchings in G. For this task, it is irrelevant what the matching property is, it is only important that $\mathcal{S}$ is a family of subsets of E.

Let L be some positive integer. Assume that we draw a weight $w(e)$ for each edge independently and uniformly at random from $\{1, \ldots, L\}$. The weight of a set $M \in \mathcal{S}$ (a matching) is defined as $w(M) = \sum_{e \in M} w(e)$.

Let M^* be an element from $\mathcal{S}$ with maximum weight (a maximum matching), i.e. $w(M) \leq M^*$ for all $M \in \mathcal{S}$. Prove that the probability that M^* is the *unique* element in $\mathcal{S}$ with weight $w(M^*)$ is at least $1 - \frac{m}{L}$. In other words, prove

$$\Pr(\exists M' \in \mathcal{S} \backslash M^* : w(M') = w(M^*)) < \frac{m}{L}.$$