## Complexity of Boolean functions SS 2018 Homework 4

14.05.2015

## Exercise 1:

Prove that each prime implicant of a monotone function  $f \in M_n$  contains only nonnegated variables.

## Exercise 2:

Prove Theorem 3.1 of the lecture.

## Exercise 3:

Let  $g = res_{\beta}(v)$  for a gate v in a monotone network  $\beta$  for the function f. Prove the following assertions:

- a) g can be replaced by the constant zero iff for all  $t \in PIM(g)$  for all monomials t' there holds  $tt' \notin PIM(f)$ .
- b) g can be replaced by the constant one iff for all functions h there hold  $gh \leq f$  implies  $h \leq f$ .