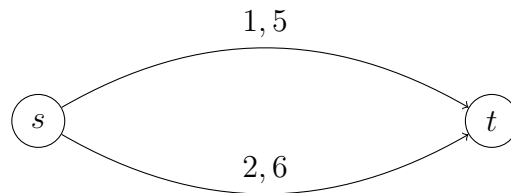


Algorithmic Game Theory and the Internet
Summer Term 2018
Exercise Set 5

Exercise 1: (1+3+2 Points)
Consider the following symmetric network congestion game with two players:



- (a) What is the Price of Anarchy and the Price of Stability of pure Nash equilibria?
- (b) What is the Price of Anarchy and the Price of Stability of mixed Nash equilibria?

Hint: First of all, determine all mixed Nash equilibria. For this purpose, start with a sentence like “Let σ be a mixed Nash equilibrium with $\sigma_1 = (\lambda_1, 1 - \lambda_1)$, $\sigma_2 = (\lambda_2, 1 - \lambda_2)$ ” and subsequently derive properties of λ_1 and λ_2 .

- (c) What is the best upper bound for the Price of Anarchy that can be shown by smoothness?

Exercise 2: (3 Points)
State for each $M \geq 1$ a network congestion game with two players such that the Price of Anarchy of pure Nash equilibria is at least M .

Exercise 3: (3 Points)
Recall *Fair Cost-Sharing Games* as congestion games such that for all resources $r \in \mathcal{R}$ the delay function can be modeled as $d_r(x) = c_r/x$ for a constant c_r . Show that fair cost sharing games with n players are $(n, 0)$ -smooth.

Exercise 4:

(4 Points)

In the lecture, we assumed the social cost is given by $cost(s) = \sum_{i \in \mathcal{N}} c_i(s)$. On this basis, we defined the Price of Anarchy which will be denoted by POA_{Eq}^{Σ} . Another reasonable definition of the social cost could be $cost(s) = \max_{i \in \mathcal{N}} c_i(s)$. Hence, we get an additional definition of the Price of Anarchy POA_{Eq}^{\max} .

State an example of a game in which $POA_{PNE}^{\Sigma} > POA_{PNE}^{\max}$ and another game for $POA_{PNE}^{\Sigma} < POA_{PNE}^{\max}$.

Exercise 5:

(4 Points)

Consider a (λ, μ) -smooth game with N players and let $s^{(1)}, \dots, s^{(T)}$ be a sequence of states such that the external regret of every player is at most $R^{(T)}$. Moreover, let s^* denote a state that minimizes the social cost. We want to upper bound the average social cost of the sequence of states. For this purpose, prove the following bound

$$\frac{1}{T} \sum_{t=1}^T cost(s^{(t)}) \leq \frac{N \cdot R^{(T)}}{(1 - \mu)T} + \frac{\lambda}{1 - \mu} cost(s^*).$$