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Due date: In the lecture on 9th July 2018, 10:15

Algorithmic Game Theory and the Internet

Summer Term 2018

Exercise Set 12

Exercise 1: (3 Points)

Consider the following instance of the house-allocation problem. There are five agents $a, \ldots, e$ and their preferences are given by:

$$a : b > d > f > e > c > a\,, \qquad b : d > a > c > e > f > b\,,$$
$$c : e > f > a > c > b > d\,, \qquad d : e > a > b > c > d > f\,,$$
$$e : f > e > c > b > d > a\,, \qquad f : d > a > b > c > f > e\,.$$

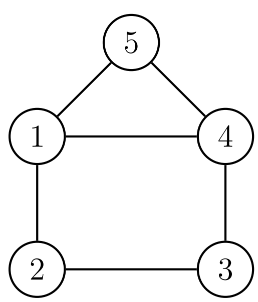
Make use of the Top Trading Cycle Algorithm in order to state an allocation π that is stable.

Exercise 2: (4+4 Points)

Consider the problem of Pairwise Kidney Exchange by Matching from Lecture 23. The graph below depicts an instance of agents (that is, patient-donor pairs) and possible pairwise exchanges.

(a) Use the mechanism of section 4 from Lecture 23 and consider agents in ascending order of agent indices (which is independent of the reports) to determine the set of maximum matchings M_5.

(b) Now, let us show that the given matching algorithm is not DSIC if the order in which the algorithm processes the agents depends on their reports. For this purpose, consider a modified algorithm that processes agents in ascending order of node degree (tie-breaking in favor of the agent with the smallest index) and verify that agent 4 can do better by misreporting in the given instance.



Exercises 3 and 4 on the next page.

Exercise 3: (5 Points)
Consider a set of n teams, each with 10 players, where each team owner has a ranking of all $10n$ players. Define a notion of *stable allocation* in this setting (as in Definition 23.1) and show how to adapt the top trading cycle algorithm to find a stable allocation. We assume that players' preferences play no role.[1]

Exercise 4: (4 Points)
A weaker notion of instability than the one discussed in Section 1 of Lecture 23 requires that no set of agents can obtain better houses than they are assigned in π by reallocating among themselves the houses allocated to them in π. Show that this follows from stability as defined in Section 1.[2]

Note: The converse does not hold. For example, if there are two agents who both prefer the same house, the only stable allocation is to give that house to its owner, but the alternative is also weakly stable.

[1] Exercise is taken from Chapter 10 of the Karlin/Peres book

[2] See footnote 1