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Due date: In the lecture on 3rd July 2019, 12:15

Algorithmic Game Theory and the Internet

Summer Term 2019

Exercise Set 12

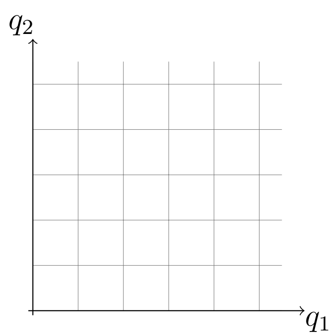
Exercise 1: (2+3 Points)

Consider three unit-demand buyers and two items with

$$v_{1,1} = 5, v_{1,2} = 3, v_{2,1} = 3, v_{2,2} = 4, v_{3,1} = 2, v_{3,2} = 2 \ .$$

(a) Determine the Walrasian price vector that is determined by the VCG mechanism.

(b) Now find *all* Walrasian price vectors q. (We know that the solution to (a) is component-wise smaller than any other such vector.) Draw these vectors in a coordinate system with axes q_1 and q_2.



Exercise 2: (4 Points)

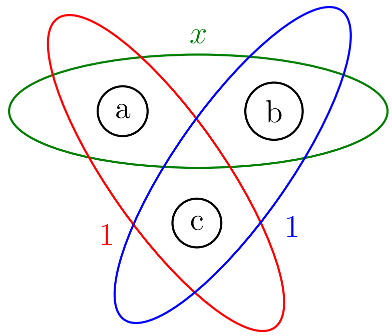
Consider m items and n unit-demand bidders. We define a generalization of Walrasian equilibria: Let S be a matching of items to bidders and $q \in \mathbb{R}^m_{\geq 0}$ be a price vector. We call the pair (q, S) an *ϵ-approximate Walrasian equilibrium* if unallocated items have price 0, every bidder i has non-negative utility $v_{i,S(i)} - q_{S(i)} \geq 0$, and every bidder receives an item within ϵ of its favorite, i.e., $v_{i,S(i)} - q_{S(i)} \geq v_{i,j} - q_j - \epsilon$ for every item j.

Prove an approximate version of the First Welfare Theorem: If (q, S) is an ϵ-approximate Walrasian equilibrium, then the social welfare of an optimal matching S^* cannot surpass the one of S by more than $\min\{m, n\} \cdot \epsilon$.

Exercises 3 and 4 on the next page.

Exercise 3: (3 Points)

Have a look at the single-minded combinatorial auction with three bidders and items a, b, c which is depicted below. State all values of $x \in \mathbb{R}_{\geq 0}$ such that there exists a Walrasian equilibrium and prove your claim.



Exercise 4: (4+4 Points)

Recall the valuation functions of single-minded bidders from Definition 12.2. Let the maximum bundle size be defined by $d = \max_{i \in \mathcal{N}} |S_i^*|$.

(a) Show that in the case of single-minded bidders with maximum bundle size d, item bidding (cf. Section 1 of Lecture 16) with first price payments is $(\frac{1}{2}, 2d)$-smooth.

Hint: In order to define deviation bids $b_{i,j}^*$, consider a welfare-maximization allocation on v. If bidder i does not get his bundle in the optimal allocation, then define $b_{i,j}^* = 0$ for all items $j \in M$. Otherwise, define $b_{i,j}^* = \frac{v_i}{2d}$ for all $j \in S_i^*$ and $b_{i,j}^* = 0$ if $j \notin S_i^*$. That is, each winner in the optimal allocation equally divides the value for his bundle among all items of the bundle and bids half of it.

(b) Now, we define prices for items as in Lecture 21 by setting

$$p_j^v = \begin{cases} \frac{1}{2d} v_i(S_i^*) & \text{if buyer } i \text{ gets item } j \text{ in optimal solution on } v \\ 0 & \text{if item } j \text{ is unassigned in optimal solution on } v \end{cases}$$

Show that using these prices in the full-information setting gives a $\frac{1}{2d}$-approximation of the optimal social welfare. (Like in Step 1 of Lecture 21)