

Problem Set 1

Please hand in your solutions for this problem set via email (roesner@cs.uni-bonn.de) or personally at Room 2.060 until *Tuesday, 16th of October*.

Problem 1

Recall that Gonzalez' algorithm chooses in each step one of the points as a center. For the k -center problem, this is a valid strategy. The k -supplier problem is a generalization of the k -center problem. It is defined on a finite metric space (X, d) , but X is not the input point set. Instead, there are two sets $P \subseteq X$ and $L \subseteq X$, and the problem is to compute a set of k centers $C \subset L$ which minimizes the maximal distance between a point in P and its cluster center. The k -center problem is a special case of the k -supplier problem where $P = L = X$.

- Show how Gonzalez' algorithm can be adjusted for the k -supplier problem.
- Show what the approximation factor of the adjusted algorithm is?

Problem 2

For an upcoming election, a city counsel wants to decide where to place voting centers. They have multiple options in public buildings such as schools and kindergartens. Opening a voting center has some fixed costs for the staff, which we assume is the same for every possible option. In order to minimize the cost, the city could open just one voting center. But this would imply that some of the potential voters live too far away from the voting center. In the board meeting two approaches were suggested. The first approach suggest that at first a reasonable budget, which decides how many voting centers we can be afforded, should be fixed. The task then is to try to place the centers in order to minimize the longest distance a voter has to go to their nearest voting center. The second approach suggest to decide what a reasonable distance for a voter to go to their voting center is and then try to minimize the number of voting centers needed such that nobody is further away from their closest voting center.

- Show how both approaches can be solved by solving a k -supplier problem.

Problem 3

Recall the Dominating Set Problem: Given a Graph $G = (V, E)$ find a minimal set of vertices $V' \subset V$ such that for each vertex $v \in V$ we have $v \in V'$ or there exists a vertex $w \in V'$ with $\{v, w\} \in E$.

- Show how an Algorithm that solves the Dominating Set Problem can be used to solve the k -center Problem.
- Can approximation algorithms for the Dominating Set Problem be used in a similar manner to solve or approximate the k -center problem?