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Due: lecture on January 8, 2019

Algorithms and Uncertainty

Winter Semester 2018/19

Exercise Set 8

Exercise 1: (5 Points)
State a no-regret algorithm for the case that $\ell_i^{(t)} \in [-\rho, \rho]$ for all i and t. Also give a bound for the regret. You should reuse algorithms and results from the lectures.

Exercise 2: (5 Points)
Wo consider a different form of feedback. After step t, the algorithm does not get to know $\ell_i^{(t)}$ for all i but a noisy version. More precisely, an adversary first fixes the sequence $\ell^{(1)}, \ldots, \ell^{(T)}$, where all costs are in $[0, 1]$. Afterwards, from this sequence $\bar{\ell}^{(1)}, \ldots, \bar{\ell}^{(T)}$ is computed, where $\bar{\ell}_i^{(t)} = \ell_i^{(t)} + \nu_i^{(t)}$ and $\nu_i^{(t)}$ is an independent random variable on $[-\epsilon, \epsilon]$ with $\mathbf{E}[\nu_i^{(t)}] = 0$.
State a no-regret algorithms and a bound for the regret. Use the previous exercise and the ideas presented in Lecture 17.

Exercise 3: (5 Points)
Show the Follow-the-Regularized-Leader with Entropical regularization in the experts setting is equivalent to the Multiplicative Weights algorithm.

Hint: It can be helpful to use a Langrange multiplier, which works in this special case as follows: For $\mathbf{x}$ to be a local optimum of F subject to $\sum_{i=1}^{d} x_i = 1$, it is necessary that there exists a $\lambda \in \mathbb{R}$ such that $\frac{\partial F}{\partial x_i}(\mathbf{x}) - \lambda = 0$ for all i.

Exercise 4: (5 Points)
Consider the following problem motivated by web search: Suppose there are T users that all search for the same keyword. There are k different results that they might be interested in. Whenever a user arrives, we display these k results in an order that we choose. Afterwards we get to know which of the k results the user was interested in and incur a cost of j if this was the j^{th} result in our order.
Model this problem as an online convex optimization problem so that Follow the (Regularized) Leader can be applied.

Happy Holidays!