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January 8, 2019
Due: lecture on January 15, 2019

Algorithms and Uncertainty

Winter Semester 2018/19

Exercise Set 9

Exercise 1: (3 Points)
Prove Observation 20.4: If R is σ-strongly convex and $f_1, f_2, \ldots$ are convex then $R + \sum_t f_t$ is σ-strongly convex.

Exercise 2: (4 Points)
We consider Online Linear Regression as introduced in the lecture on December 18. Recall that

$$f_t(w_1, w_2) = \left(w_1 x^{(t)} + w_2 - y^{(t)}\right)^2 \quad .$$

Derive a regret bound for Follow-the-Regularized-Leader with Euclidean regularization under the assumption that $|x^{(t)}|, |y^{(t)}| \leq 1$ for all t and $S = \{\mathbf{w} \in \mathbb{R}^2 \mid \|\mathbf{w}\|_2 \leq r\}$.

Exercise 3: (3 Points)
Derive a regret bound for Follow-the-Regularized-Leader if the Lipschitz constant depends on the time step, that is,

$$f_t(\mathbf{u}) - f_t(\mathbf{v}) \leq L_t \|\mathbf{u} - \mathbf{v}\| \quad \text{for all } \mathbf{u}, \mathbf{v} \in S \quad .$$

Exercise 4: (3 Points)
Consider a finite set X. Show that in this case every hypothesis class $\mathcal{H}$ is PAC learnable. Use results from the lecture on January 8 but not from one on January 10.

Exercise 5: (4+3 Points)
Let $X = \mathbb{R}$ and let $\mathcal{H}$ be the hypotheses of the for $h(x) = 1$ for $x \in [a, b]$, $h(x) = 0$ otherwise. Give the growth function $\mathcal{H}[m]$ and prove your claim as follows.

(a) Show that for every set S with $|S| = m$ it holds that $\mathcal{H}[S] \leq \mathcal{H}[m]$.

(b) For every m, give a set S with $|S| = m$ such that $\mathcal{H}[S] = \mathcal{H}[m]$. Justify your claim.