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## Advanced Algorithms

WS 2017/18

### Homework 1

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#### Exercise 1:

Let us consider depth-first search of a directed graph. Let  $Q$  be the stack which is used for the organization of the search. Show that during the depth-first search the stack  $Q$  always contains a simple path from the start node to the top node of  $Q$ .

#### Exercise 2:

- a) Develop an algorithm which decides in linear time if a given graph  $G = (V, E)$  is bipartite or not.
- b) Develop an algorithm which decides if a given graph  $G = (V, E)$  is bipartite or not after the deletion of one edge. What is the run time of your algorithm? Can you give a linear time algorithm for this problem?

#### Exercise 3:

Construct a graph which contains a maximum matching of size ten and a maximal matching of size five.

#### Exercise 4:

Let  $G = (V, E)$  be an undirected graph,  $M$  a maximal and  $M'$  a maximum matching of  $G$ . Prove  $|M| \geq \frac{|M'|}{2}$ .