

Advanced Algorithms

WS 2017/18

Homework 2

23.10.2017

Exercise 1:

A vertex cover X of a graph $G = (V, E)$ contains for every edge $e \in E$ at least one of its end nodes. A vertex cover $X \subseteq V$ is *minimum* if $|X| \leq |X'|$ for all vertex covers $X' \subseteq V$. Show that in bipartite graphs the size of a maximum matching is equal to the size of a minimum vertex cover.

Exercise 2:

- a) Work out the algorithm for the computation of a maximum weighted matching in bipartite graphs in detail.
- b) Show that the algorithm MAXWEIMATCHING can be implemented such that its run time is $O(n^3)$ where $n = |A \cup B|$.

Exercise 3:

Construct a weighted undirected graph with the property that a matching of maximum weight is not a matching of maximum size.

Exercise 4:

Let M_1 and M_2 be two distinct matchings of a graph G such that $|M_1| = r$, $|M_2| = s$ and $r \leq s$. Show that there exist two distinct matchings M'_1 and M'_2 such that

$$\left\lfloor \frac{s+r}{2} \right\rfloor \leq |M'_1| \leq |M'_2| \leq \left\lceil \frac{s+r}{2} \right\rceil.$$