MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20
Assignment 12

Deadline: **21** January before noon (To be discussed: **21/22**. January 2020)

1 Maximum level in an arrangement

Given a set L of n lines in the plane, the level of a point $x \in \mathbb{R}^2$ is the number of lines lying strictly above x. Give an $O(n \log n)$ time algorithm to compute the maximum level of any vertex in the arrangement $\mathcal{A}(L)$, where L is not necessarily in general position.

2 Vapnik-Chervonenkis dimension

1. Consider the range space $S = (X, \mathcal{R})$. The *complement* of S is defined as the space $\overline{S} = (X, \overline{\mathcal{R}})$, where $\overline{\mathcal{R}} = \{X \setminus r \mid r \in \mathcal{R}\}$. Show how the VC dimension of $\overline{S}$ is related to the VC dimension of S.

2. Let $S_1 = (X, \mathcal{R}_1)$, $S_2 = (X, \mathcal{R}_2)$ be two range spaces with VC dimension δ_1 and δ_2 respectively. Let $\hat{\mathcal{R}} = \{r_1 \cap r_2 \mid r_1 \in \mathcal{R}_1, r_2 \in \mathcal{R}_2\}$. Show that the VC dimension of $(X, \hat{\mathcal{R}})$ is $O((\delta_1 + \delta_2)\log(\delta_1 + \delta_2))$.

3. Consider a range space $S = (X, \mathcal{R})$, where X is a finite subset of $\mathbb{R}^2$ and $\mathcal{R} = \{\Delta \cap X \mid \Delta \text{ is a triangle in } \mathbb{R}^2\}$. Show an upper bound on the VC dimension of S.

4. Consider a range space $S = (X, \mathcal{R})$, where X is a finite subset of $\mathbb{R}^2$ and $\mathcal{R} = \{\Pi_k \cap X \mid \Pi_k \text{ is a convex } k\text{-gon in } \mathbb{R}^2\}$, for a given integer k. Show an upper bound on the VC dimension of S.