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Due date: January 13, 2021 at noon

Algorithmic Game Theory

Winter Term 2020/21

Exercise Set 8

Exercise 1: (4 Points)

Show that if a mechanism is (λ, μ)-smooth and players have the possibility to withdraw from the mechanism then $PoA_{\mathsf{CCE}} \leq \frac{\max\{1,\mu\}}{\lambda}$.

Exercise 2: (3 Points)

Recall the auction of k identical items from the previous exercise sets. Bidder i has value v_i if he/she gets one of the items, 0 otherwise.
We define a mechanism as follows: the bidders who reported the k highest bids win an item. Each of them has to pay their respective bids.
Show that if losers (i.e. bidders who do not get any item) pay their bids, this mechanism is $(\frac{1}{2}, 2)$-smooth.

Remark: Notice the difference concerning the losers' payments compared to Exercise 2 from the tutorials. Now, all bidders pay their respective bids. We have already seen such a payment rule in Homework Set 7, Exercise 3 when considering a single-item all-pay auction.

Exercise 3: (6 Points)

We work in the setup of combinatorial auctions with m (possibly heterogeneous) items M. Bidders report bids for items and afterwards each item is sold in a separate second-price auction (item bidding). Prove the following theorem.

Theorem. Consider a pure Nash equilibrium b of item bidding with second-price payments and unit-demand bidders. Let $X_1, \ldots, X_n$ be the resulting allocation. If for all bidders i we have $\sum_{j \in X_i} b_{i,j} \leq v_i(X_i)$ (weak no-overbidding), then $\sum_{i \in N} v_i(X_i) \geq \frac{1}{2} OPT(v)$.

Hint: Make use of the following deviation bids: Consider the welfare-maximizing allocation on v. Let j_i be the item that is assigned to bidder i in this allocation. If i does not get any item, set j_i to $\perp$. Set $b^*_{i,j} = v_{i,j}$ if $j = j_i$ and 0 otherwise. Now, derive a proof in the spirit of Theorem 16.2.