Discrete and Computational Geometry, WS1415
Exercise Sheet "3": Randomized Algorithms for Geometric Structures III
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 4th of November 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 7: L_1 **Voronoi diagram.** **(4 Points)**

For two points $p = (x_p, y_p)$ and $q = (x_q, y_q)$ in the plane, the L_1 distance $d_1(p, q)$ between p and q is $|x_p - x_q| + |y_p - y_q|$. Given a set S of n point sites, the L_1 Voronoi diagram $V(S)$ of S is a planar subdivision such that all points in a region share the same nearest site in S under the L_1 distance.

1. Describe in which situation the bisector $B_1(p, q) = \{r \in \mathbb{R}^2 \mid d_1(r, p) = d_1(r, q)\}$ between p and q has two-dimensional faces.
2. Describe the possible shapes of an L_1 bisector between two points if it has no two-dimensional face.
3. As mentioned in the lecture, we can generate n identical paraboloids or n identical cones with apices respectively on the n points such that seeing at $(0, 0, -\infty)$ upward will find the Euclidean Voronoi diagram. Please describe the shape of cones for the L_1 Voronoi diagram.

Exercise 8: Triangulation (History Graph) (4 Points)

Given a set N of n points in the plane, a triangulation $H(N)$ of N is a maximal planar straight-light graph, i.e., every edge is a straight-line segment, and no edge can be added to main the planarity. Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the history graph. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}. (Hint: You can use the three dummy points as Exercise 4.)

1. Describe the parent and child relation in the history graph.
2. Describe the insertion of S_{i+1} using the history graph.
3. Prove the expected cost of inserting S^{i+1} to be $O(\log i)$ and the expected cost of construction $T(N)$ to be $O(n \log n)$

Exercise 9: Planar Convex Hull by History Graph (4 Points)

Given a set N of n points in the plane, a convex hull $H(N)$ of N is a minimal convex polygon containing N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^3), H(N^4), \ldots, H(N^n)$ iteratively using the history graph. In other words, for $i \geq 3$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}.

1. Describe the parent and child relation in the history graph.
2. Describe the insertion of S_{i+1} using the history graph.
3. Prove the expected cost of inserting S^{i+1} to be $O(\log i)$ and the expected cost of construction $T(N)$ to be $O(n \log n)$