Institut für Informatik
Prof. Dr. Heiko Röglin
Dr. Melanie Schmidt

Randomized Algorithms and
Probabilistic Analysis
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Problem Set 0

Problem 1

Let $(\Omega, \Pr)$ be a discrete probability space and let $A, B \in 2^\Omega$ be events that are independent. Show that $\bar{A}$ and $\bar{B}$ are independent.

Problem 2

We flip a fair coin n times. We are interested in sequences tosses that all come up heads. For simplicity, let n be a power of two.

- Show that the probability that we see a sequence of $1 + \log n$ heads is at most $1/2$.
- Show that the probability to see a sequence of more than $1 + \log n$ heads decreases exponentially. To do that, find an upper bound on the probability for a sequence with $k + \log n$ heads that decreases exponentially in k.

Problem 3

In this task, we want to cut a graph $G = (V, E)$ into r pieces instead of cutting it into two pieces as in the lecture. We say that r disjoint subsets $V_1, \ldots, V_r$ with $V = \cup_{i=1}^r V_i$ are an r-*cut* of G. We pay for all edges between these subsets, our cost is: $\frac{1}{2}(|\delta(V_1)| + |\delta(V_2)| + \ldots + |\delta(V_r)|)$. We want to find an r-cut with minimum cost.

Generalize Karger's contract algorithm such that it finds an r-cut and give a lower bound on the probability that it outputs a minimum r-cut.