

Discrete and Computational Geometry, WS1516  
Exercise Sheet “4”: Chan’s Technique and Detours  
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until **Wednesday 25th of November, 12:00 pm**.*
- *There is a letterbox in front of Room E.01 in the LBH building.*
- *You may work in groups of at most two participants.*

**Exercise 9: Constants in Chan’s method (4 Points)**

We would like to find out how large the constants in the main lemma of Chan’s randomized technique might become. We refer to the application of computing the detour of a polygonal chain. For the detour-of-the-chain computation we choose  $\alpha = 1$  and  $r = 4$  for a decomposition as suggested. If the decision algorithm takes  $O(f(n))$  time, the randomized optimization algorithm takes  $R \times f(n)$  expected time for a constant  $R$ . Please analyze what  $R$  would be in the following way.

1. Assume that the decision algorithm runs in  $R' \times n \log n$  time.
2. Choose an  $\epsilon$  so that the precondition of Chan’s technique will be satisfied, e.g.,  $\frac{n \log n}{n^\epsilon}$  monotone increases in  $n$  and  $(\ln r + 1)\alpha^\epsilon < 1$ .
3. How many recursion steps  $l$  have to be done for your choice of  $\epsilon$ ?
4. Express constant  $R$  in terms of precise values of  $l$ ,  $\alpha$  and  $r$  and the variable parameter  $R$ .

**Exercise 10: The Decomposition of a Polygonal Chain (4 Points)**

Consider a polygonal chain  $C$  with  $n$  polygonal vertices, and let  $V$  be the set of polygonal vertices of  $C$ . For any two points  $p, q \in C$ , the detour  $\delta_C(p, q)$  between  $p$  and  $q$  in  $C$  is  $\frac{|C_p^q|}{|\overline{pq}|}$ , where  $C_p^q$  is the simple path between  $p$  and  $q$  in  $C$ , and the detour  $\delta_C$  of  $C$  is  $\max_{p, q \in C} \delta_C(p, q)$ . Let  $W$  be a subset of  $V$ , and let  $Q$  be a subchain of  $C$ . Furthermore, Let  $\delta_C(W, Q)$  be  $\max_{p \in W, q \in Q} \delta_C(p, q)$ , and let  $\delta_C^*(W, Q)$  be  $\sup_{(p, q) \in W \times Q, \overline{pq} \cap Q = \emptyset} \delta_C(p, q)$ .

- Please give an example in which there exists a pair of points,  $p \in W$  and  $q \in Q$  such that  $\delta_C(p, q) = \delta_C(W, Q)$  but  $\overline{pq}$  intersects  $C$ .
- please prove that if  $\delta_C(W, Q) = \delta_C$ ,  $\delta_C(W, Q) = \delta_C^*(W, Q)$ .