Institut für Informatik
Prof. Dr. Heiko Röglin
Dr. Melanie Schmidt

UNIVERSITÄT BONN

Randomized Algorithms and
Probabilistic Analysis
Summer 2018

Problem Set 2

Please hand in your solutions until Monday, April 23rd.

Problem 1

Alice tells Bob a new game. She has three six-sided dice. The dice are fair (all sides come up with the same probability), but they do not have the standard numbering. Instead, they have the following numbers:

- die A: 1,1,6,6,8,8
- die B: 2,2,4,4,9,9
- die C: 3,3,5,5,7,7

Alice explains that she lets Bob pick a die first to give him an advantage. Then she will pick a die. Then they roll their dice, and the player with the higher number wins. After playing a while, Bob thinks he is unlucky because Alice wins more than him. Is Bob unlucky or does Alice have a higher win chance? Alice uses the following strategy:

- If Bob picks A, she picks B.
- If Bob picks B, she picks C.
- If Bob picks C, she picks A.

Problem 2

Let `ALG` be a randomized algorithm with running time $\Theta(n^3 + n + \sqrt{n})$ that outputs an optimal solution (for an unspecified optimization problem) with probability at least $\frac{1}{\sqrt{n}\log n}$. Give a number ℓ of independent repetitions such that repeating `ALG` ℓ times and returning the best solution results in an algorithm with success probability at least $1 - \frac{1}{n^7}$. What is the running time of the resulting algorithm?

Problem 3

In a given lecture at the university 63% of the students regularly attended the tutorials(and prepared the exercises) 87% of them passed the exam with a good(or very good) grade. In total 67% of the students passed the exam with a good(or very good) grade. What is the probability that:

- a student who passes the exam with a good grade did not attend the tutorials;

- a student who does not pass the exam with a good grade regularly attended the tutorials;
- a student who did not attend the tutorials passes the exam with a good grade.

Problem 4

You purchased a certain product. The manual states that the lifetime T of the product, defined as the number of years until it breaks down, satisfies

$$P(T \geq t) = e^{-\frac{t}{7}}, \text{ for all } t \geq 0.$$

For example, the probability that the product lasts more than (or equal to) 3 years is $P(T \geq 3) = e^{-\frac{3}{7}} = 0.6514$. You use it for tree years without any problems. What is the probability that it breaks down in the fourth or fifth year?