

## Problem Set 3

Please hand in your until Monday April 30th.

### Problem 1

1. A lively monkey types  $26^6 \cdot 42 + 5$  letters (=499017796 letters) on a keyboard. We assume that the keyboard has only upper-case letters and that each of the 26 letters is chosen uniformly at random. What is the expected number of times that the word RANDOM appears?
2. We flip a fair coin  $n + \log_2 n$  times, assume that  $n$  is a power of two. We get a sequence  $x_1, x_2, \dots, x_{n+\log_2 n}$  with  $x_i \in \{H, T\}$ . We say that  $x_i, \dots, x_{i+\ell-1}$  is an  $\ell$ -sequence if  $x_i = x_{i+1} = \dots = x_{i+\ell-1}$  (all heads or all tails). What is the expected number of  $\ell$ -sequences for  $\ell = 1 + \log_2 n$ ?

### Problem 2

As in the proof of Theorem 1.14 (success probability of Karger's Contract algorithm) let  $A_i$  be the event that the algorithm contracts a good edge in iteration  $i$ . Show or disprove that  $A_i$  and  $A_j$  are generally independent for  $i, j \in \{1, \dots, n-1\}$ .

### Problem 3

In this task, we want to cut a graph  $G = (V, E)$  into  $r$  pieces instead of cutting it into two pieces as in the lecture. The parameter  $r \in \mathbb{N}$  is a constant. We say that  $r$  disjoint subsets  $V_1, \dots, V_r$  with  $V = \cup_{i=1}^r V_i$  are an  $r$ -cut of  $G$ . We pay for all edges between these subsets, our cost is:  $\frac{1}{2}(|\delta(V_1)| + |\delta(V_2)| + \dots + |\delta(V_r)|)$ . We want to find an  $r$ -cut with minimum cost.

Generalize Karger's Contract algorithm such that it finds a minimum  $r$ -cut with probability  $\Theta(1/n^{3r})$ .

### Problem 4

What is the running time of the FastCut algorithm (without repetitions) when we set  $t := (3/4)n$ ? You may use the 'master theorem' (this theorem is explained in many books and lecture notes, see for example the notes from Avrim/Manuel Blum's course at <https://www.cs.cmu.edu/~avrim/451f11/lectures/lect0901.pdf>).

***Master Theorem***

The recurrence

$$T(n) = aT(n/b) + cn^k$$

$$T(1) = c,$$

where  $a$ ,  $b$ ,  $c$  and  $k$  are all constants, solves to:

$$T(n) \in \Theta(n^k) \text{ if } a < b^k$$

$$T(n) \in \Theta(n^k \log n) \text{ if } a = b^k$$

$$T(n) \in \Theta(n^{\log_b a}) \text{ if } a > b^k.$$