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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 10

Problem 1

We agree to try to meet between 12 and 1 for lunch at our favorite sandwich shop. Because of our busy schedules, neither of us is sure when we'll arrive; we assume that, for each of us, our arrival time is uniformly distributed over the hour. So that neither of us has to wait too long, we agree that we will each wait exactly 15 minutes for the other to arrive, and then leave. What is the probability we actually meet each other for lunch?

Problem 2

Consider the following *product knapsack problem*. Given n objects with deterministic weights $w_1, \ldots, w_n \in [0,1]$, a capacity W and deterministic profits $p_1, \ldots, p_n \in \mathbb{R}^{\geq 1}$, find a solution $x \in \{0,1\}^n$ that maximizes the product

$$p(x) = \prod_{i:x_i=1} p_i$$

of the profits of the chosen items under the constraint that $w^t x \leq W$. Adapt the Nemhauser-Ullmann algorithm to the product knapsack problem and argue that the adaptation computes an optimal solution to this problem.

Problem 3

We discuss bad examples for the core algorithm. In the following, you may use fractional weights and profits.

1. The idea behind the core algorithm is the hope that the optimal solution for the fractional knapsack problem as computed by the greedy algorithm on page 105, $\bar{x}$, typically differs from an optimal solution in only in a few items, and that these items lie in the core. Give an example where, except for the break item, no item that is in $\bar{x}$ is contained in any optimal solution x^*.
2. The expanding core algorithm estimates the additive integrality gap Γ by iteratively increasing it. Give an example where it increases the estimator $n-1$ times and finally ends up with all items in the core.

Problem 4

Consider an arbitrary binary optimization problem with linear objective $c^T x$ and solution set $\mathcal{S} \subseteq \{0,1\}^n$ as discussed in Chapter 7. Recall that the winner gap Δ is defined as

$$\Delta := cx^* - cx^{**}$$

where x^* is an arbitrary optimal solution and x^{**} is a solution that is optimal amongst all solutions in $\{x \in \mathcal{S} \mid x \neq x^*\}$. Find better upper bounds on $\Pr(\Delta \leq \epsilon)$ than the bound provided by Lemma 7.3 for the following scenarios:

1. The c_i are ϕ-perturbed numbers from $[0,1]$ (instead of $[-1,1]$).

   Show that $\Pr(\Delta \leq \epsilon) \leq n\phi\epsilon$.

2. The c_i are numbers from $[1,e]$ that are chosen independently from the distribution with the density

   $$f(x) = \begin{cases} \frac{1}{x} & \text{for all } x \in [1,e] \\ 0 & \text{else.} \end{cases}$$

   Show that $\Pr(\Delta \leq \epsilon) \leq n\ln(1+2\epsilon)$.