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## Complexity of Boolean functions

SS 2018

### Homework 6

28.05.2018

#### Exercise 1:

Consider the proof of Theorem 3.8 of the lecture.

- a) Show that setting  $a_0$  to 0 eliminates those  $s_0$   $\wedge$ -gates where the outputs  $f_i$  with  $f_i = a_0 b_j$  for a  $j$  are computed.
- b) Show that the property “ $P$  contains no  $\vee$ -gate  $v_l$  with  $a_i b_j \leq \text{res}_\beta(v_l)$  for an  $i \neq 0$  and some  $j$ ” implies that  $f_k$  does not contain two prime implicants.

#### Exercise 2:

Consider the construction of the network  $\beta_1$  from the network  $\beta_0$  for the convolution as presented in the lecture. Prove that after the realization of an  $\wedge$ -gate  $u$ , the properties i) – iii) are fulfilled for the network  $\delta_u$ .