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Due date: In the lecture on 7th May 2018, 10:15

Algorithmic Game Theory and the Internet

Summer Term 2018

Exercise Set 4

Exercise 1: (2 Points)
Let p, p' be coarse correlated equilibria of a cost-minimization game Γ. Prove that any convex combination of the distributions p and p' yields also coarse correlated equilibrium of Γ (i.e., any distribution $q := \lambda p + (1 - \lambda)p'$ for a $\lambda \in [0, 1]$).

Exercise 2: (2 Points)
Consider the mentioned hierarchy of equilibrium concepts from lecture 6. Show that every correlated equilibrium is also a coarse correlated equilibrium.

Exercise 3: (3 Points)
State an example of a sequence of probability distributions $p^{(t)}$ over strategies and cost vectors $\ell^{(t)}$ such that the player's external regret is negative.

Exercise 4: (4+4 Points)
Consider the following regret-minimization-algorithm.

Greedy

- Set $p_1^1 = 1$ and $p_j^1 = 0$ for all $j \neq 1$.
- In each round $t = 1, \ldots, T$:
  Let $L_{min}^t = \min_{i \in N} L_i^t$ and $S^t = \{i \in N \mid L_i^t = L_{min}^t\}$.
  Set $p_i^{t+1} = 1$ for $i = \min S^t$ and $p_j^{t+1} = 0$ otherwise.

(a) Show that the costs of Greedy are at most $N \cdot L_{min}^T + (N - 1)$.

(b) State a scheme for an example such that the stated upper bound of (a) is tight for an infinite number of values T.

Exercise 5 on the next page.

Exercise 5: (1+2+2 Points)

In the lecture we presented the Multiplicative-Weights Algorithm (MW) as an example for a no-external-regret algorithm with an a priori known and fixed time horizon T. Now, we want to analyse a modification of this algorithm that deals with unknown time horizons. For this purpose,

(a) state a no-external-regret algorithm which does not need the parameter T.

Hint: Use the algorithm of the lecture as a subroutine (no need to analyse it again). Initially, assume $T = 1$ and make use of the subroutine. Once a subroutine ends, double the parameter T and restart the subroutine.

(b) What is the regret of a single phase of this algorithm (i.e., whenever the subroutine ends)?

(c) Show that the external regret of the modified algorithm is at most $O(\sqrt{T \log N})$.