

Algorithmic Game Theory and the Internet

Summer Term 2018

Exercise Set 8

Exercise 1: (2 Points)

Recall the auction of k identical items from Exercise Set 6. Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i . Otherwise, that is, if she does not get an item, she has a value of 0. Make use of the VCG-results from the lecture in order to design a truthful mechanism for this auction. For this purpose, explicitly state the function f and calculate the payment rule p .

Exercise 2: (3+3 Points)

- (a) In the lecture, we have used a greedy algorithm in order to get a 2-approximation for the edge weighted bipartite matching problem. Show that the solutions of the algorithm are monotone in each component. I.e., if e is an edge chosen by the algorithm, then e will be also chosen if its weight is raised provided that all other weights remain unchanged.
- (b) Consider a single-parameter problem and let f be the function that maximizes $\sum_i b_i x_i$ among all $x \in X$ (declared welfare). Show that f is monotone.

Exercise 3: (4 Points)

As seen in the lecture, let $f: V \rightarrow X$ be a function that maximizes declared welfare, i.e., $f(b) \in \arg \max_{x \in X} \sum_i b_i(x)$ for all $b \in V$. For each i , let h_i be an arbitrary function $b_{-i} \mapsto h_i(b_{-i})$ which does not depend on b_i . We define a mechanism $M = (f, p)$ by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j \neq i} b_j(f(b)) .$$

Prove that M is a truthful mechanism.

Exercise 4: (4+4 Points)

Consider the following *Procurement Auction*. It's being attempted to buy a certain item. There are n vendors who are able to manufacture the wanted item. Vendor i incurs a cost of c_i for crafting the item. Now, the vendors are asked to state their costs for crafting the item and a vendor with lowest cost shall be chosen. The latter potentially gets a payment for it. The stated problem can be formalized by the general model of the lecture: Each vendor i is interpreted as a bidder who has negative valuation v_i , if he/she is chosen to craft the item, that is, $v_i(x) = -c_i$, if i is chosen in x .

- (a) The results of the lecture concerning VCG are applicable in this situation. Make use of them in order to state a truthful mechanism. Note that this mechanism won't be *individually rational*.
- (b) Make use of the results from Exercise 3 in order to modify the mechanism to be individually rational.