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Due date: In the lecture on 18th June 2018, 10:15

Algorithmic Game Theory and the Internet

Summer Term 2018

Exercise Set 9

Exercise 1: (2+1+3 Points)

Consider the following single-item auction: Each bidder reports a bid $b_i > 0$. The bidder with the highest bid wins the item and pays *half* his bid.

(a) Show that if we only consider two bidders and valuations are drawn uniformly from $[0, 1]$, then truthful bidding is a Bayes-Nash equilibrium.

(b) Show that this mechanism is not dominant-strategy incentive compatible.

(c) Show that it is $(\frac{1}{2}, 1)$-smooth.

Exercise 2: (4 Points)

Show that if a mechanism is (λ, μ)-smooth and players have the possibility to withdraw from the mechanism then $PoA_{\mathsf{CCE}} \leq \frac{\max\{1,\mu\}}{\lambda}$.

Exercise 3: (4 Points)

Be inspired by the steps of Section 2 in the notes of Lecture 16 to derive the symmetric Bayes-Nash equilibrium of an all-pay auction with n bidders and identical distributions.

Exercise 4: (3+3 Points)

Let us consider an auction of k identical items. Bidder i has value v_i if he gets at least one of the items, 0 otherwise. We define a mechanism as follows: the bidders who reported the k highest bids win an item. Each of them has to pay their respective bids.

(a) Show that if losers do not pay anything, this mechanism is $(\frac{1}{2}, 1)$-smooth.

(b) Show that if losers pay their bids, this mechanism is $(\frac{1}{2}, 2)$-smooth.