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Due date: In the lecture on 5th June 2019, 12:15

Algorithmic Game Theory and the Internet
Summer Term 2019
Exercise Set 9

Exercise 1: (2+1+3 Points)
Consider the following single-item auction: Each bidder reports a bid $b_i \geq 0$. The bidder with the highest bid wins the item and pays *half* his bid.

(a) Show that if we only consider two bidders and valuations are drawn uniformly from $[0, 1]$, then truthful bidding is a Bayes-Nash equilibrium.

(b) Show that this mechanism is not dominant-strategy incentive compatible.

(c) Show that it is $(\frac{1}{2}, 1)$-smooth.

Exercise 2: (4 Points)
Show that if a mechanism is (λ, μ)-smooth and players have the possibility to withdraw from the mechanism then $PoA_{\mathsf{CCE}} \leq \frac{\max\{1, \mu\}}{\lambda}$.

Exercise 3: (4 Points)
An *all-pay auction* is a single-item auction defined in almost the same manner as a first-price auction: Each bidder reports a bid $b_i \geq 0$. The bidder with the highest bid wins the item. However, every bidder must pay their own bid regardless of whether they win the prize or not.

Be inspired by the steps of Section 2 in the notes of Lecture 14 to derive the symmetric Bayes-Nash equilibrium of an all-pay auction with n bidders and identical distributions.

Exercise 4: (3+3 Points)
Recall the auction of k identical items from the previous exercise sets. Bidder i has value v_i if he/she gets at least one of the items, 0 otherwise. We define a mechanism as follows: the bidders who reported the k highest bids win an item. Each of them has to pay their respective bids.

(a) Show that if losers do not pay anything, this mechanism is $(\frac{1}{2}, 1)$-smooth.

(b) Show that if losers pay their bids, this mechanism is $(\frac{1}{2}, 2)$-smooth.