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Due: July 8, 2020 at noon

Algorithms and Uncertainty

Summer Term 2020

Exercise Set 9

Exercise 1: (2+2 Points)

Let each $l_i^{(t)} \in \{0,1\}$. We consider the following Greedy algorithm. In each step t, the algorithm selects I_t which satisfies $I_t = \arg\min_{i \in [n]} L_i^{(t-1)}$, i.e. the expert with the best cumulative cost so far (ties are broken adversarially).

(a) Show that $L_{\text{Alg}}^{(T)} \leq n \cdot \min_i L_i^{(T)} + (n-1)$

(b) Is the result of (a) surprising? Argue by the use of an appropriate lower bound.

Exercise 2: (5 Points)

State a no-regret algorithm for the case that $\ell_i^{(t)} \in [-\rho, \rho]$ for all i and t. Also give a bound for the regret. You should reuse algorithms and results from the lectures.
Hint: Chapter 4 of lecture 16 might be helpful.

Exercise 3: (5 Points)

We consider a different form of feedback. After step t, the algorithm does not get to know $\ell_i^{(t)}$ for all i but a noisy version. More precisely, an adversary first fixes the sequence $\ell^{(1)}, \ldots, \ell^{(T)}$, where all costs are in $[0,1]$. Afterwards, from this sequence $\bar{\ell}^{(1)}, \ldots, \bar{\ell}^{(T)}$ is computed, where $\bar{\ell}_i^{(t)} = \ell_i^{(t)} + \nu_i^{(t)}$ and $\nu_i^{(t)}$ is an independent random variable on $[-\epsilon, \epsilon]$ with $\mathbf{E}[\nu_i^{(t)}] = 0$.
State a no-regret algorithms and a bound for the regret. Use the previous exercise and the ideas presented in lecture 17.

Exercise 4: (3 Points)

In the lecture, we used that $\mathbf{E}\left[\min_i \sum_{t=1}^T \ell_i^{(t)}\right] \leq \min_i \mathbf{E}\left[\sum_{t=1}^T \ell_i^{(t)}\right]$ or $\mathbf{E}\left[\max_i \sum_{t=1}^T r_i^{(t)}\right] \geq \max_i \mathbf{E}\left[\sum_{t=1}^T r_i^{(t)}\right]$ respectively. Give a proof of this inequality.