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Due: June 9, 2021 at noon

Algorithms and Uncertainty

Summer Term 2021

Exercise Set 5

As there is the midterm break next week and another public holiday on June 3 (Fronleichnam), there will be no tutorials in the next two weeks. We will discuss this homework set in the tutorials on June 10.

Exercise 1: (2 Points)
Consider the following distribution for the prize of box i: the prize v_i is equal to w_i with probability q_i and is 0 else. Compute the fair cap.

Exercise 2: (3 Points)
In order to generalize the Pandora's Box setup from the lecture, suppose we would like to match people $i \in [n]$ to boxes $j \in [m]$ (each person can take at most one prize home). We know that person i's value v_{ij} for the prize in box j is independently drawn from a distribution $\mathcal{D}_{ij}$, but it costs c_{ij} to inspect the exact value of the box v_{ij}. Consider A_{ij}, I_{ij}, σ_{ij}, κ_{ij} and b_{ij} to be the corresponding generalizations of the variables introduced in the lecture.

Show that for any policy π, the expected value is given by

$$V(\pi) = \sum_{i,j} \mathbf{E}\left[A_{ij}\kappa_{ij} - (I_{ij} - A_{ij})b_{ij}\right] \quad .$$

Exercise 3: (3+4 Points)
We extend the problem of opening boxes from Lecture 13. We are still allowed to open k boxes, but now, we may keep up to ℓ prizes instead of only one.

(a) Derive a linear program such that the expected reward of any adaptive policy is upper-bounded by the value of the optimal LP solution. Give a proof.

(b) Show that the adaptivity gap is still at most 8.