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Algorithms and Uncertainty

Summer Term 2021

Exercise Set 6

Exercise 1: (3+3 Points)

As in Tutorial 6, Task 2, we consider the Stochastic Vertex Cover problem. The edge set $A \subseteq E$ is uncertain, but drawn from a known probability distribution. The probability that the edge set is $A \subseteq E$ is given by p_A. Our goal is to compute a Vertex Cover of minimum cost for the graph $G = (V, A)$. Before A is revealed, we have to pay c_v^{I} for v, afterwards $c_v^{\mathrm{II}} \geq c_v^{\mathrm{I}}$.

In order to compute a feasible policy, we use the following algorithm which uses an optimal LP solution (x^*, y^*) of the LP from Tutorial 6, Task 2:

1. In the first stage, pick all vertices for which $x_v^* \geq \frac{1}{4}$.
2. In the second stage, when knowing A, pick all vertices for which $y_{A,v}^* \geq \frac{1}{4}$.

(a) Show that this algorithm always computes a feasible policy.

(b) Show that the expected cost of the computed policy is at most 4-times the expected cost of the optimal policy.

Exercise 2: (3+4+2 Points)

We consider the following modified version of the Boosted Sampling algorithm for Stochastic Steiner Tree from the lecture. The only difference is that it uses ℓ sets $S_1, \ldots, S_\ell$ in the first phase. Show that the approximation guarantee is $\max\{2(1 + \frac{\lambda}{\ell+1}), 2(\frac{\ell}{\lambda} + 1)\}$. To this end, consider the following tasks concerning the cost of the respective phases.

(a) Give an appropriate analysis for the first phase.

(b) Give an appropriate analysis for the second phase.

(c) Combine both results to derive the desired approximation guarantee.