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Due date: May 2, 2023 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2023

Exercise Set 4

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (3+3 Points)

We want to derive properties of the sets of correlated and coarse correlated equilibria.

(a) Show that the set of correlated equilibria of a cost-minimization game Γ is convex, i.e. for two correlated equilibria p, p' and $\lambda \in [0,1]$, also $\lambda p + (1-\lambda)p'$ is a correlated equilibrium.

(b) Show that every correlated equilibrium is also a coarse correlated equilibrium.

Exercise 2: (4+4 Points)

Consider the following regret-minimization-algorithm.

Greedy

- Set $p_1^1 = 1$ and $p_j^1 = 0$ for all $j \neq 1$.
- In each round $t = 1, \dots, T$:

  Let $L_{min}^t = \min_{i \in N} L_i^t$ for $L_i^t = \sum_{t' \leq t} \ell_i^{(t')}$ and $S^t = \{i \in N \mid L_i^t = L_{min}^t\}$.
  Set $p_i^{t+1} = 1$ for $i = \min S^t$ and $p_j^{t+1} = 0$ otherwise.

You can assume that $\ell_i^{(t)} \in \{0, 1\}$ for all i and t.

(a) Show that the costs of Greedy are at most $N \cdot L_{min}^T + (N-1)$.

(b) State a scheme for an example such that the stated upper bound of (a) is tight for an infinite number of values T.

Exercise 3: (5 Points)

We consider the Multiplicative-Weights Algorithm with a slightly modified update rule. Instead of using $w_i^{(t+1)} = w_i^{(t)} \cdot (1-\eta)^{\ell_i^{(t)}}$, we now use $w_i^{(t+1)} = w_i^{(t)} \cdot \left(1 - \eta \cdot \ell_i^{(t)}\right)$. Prove a statement as in Proposition 7.7. for this modified update rule.