

## Algorithmic Game Theory

Summer Term 2023

### Exercise Set 12

*If you want to hand in your solutions for this problem set, please send them via email to [anna.heuser@uni-bonn.de](mailto:anna.heuser@uni-bonn.de) by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

*If you would like to present one of the solutions in class, please also send an email to [anna.heuser@uni-bonn.de](mailto:anna.heuser@uni-bonn.de) containing the **task** which you would like to present and in **which of the tutorials** you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

**Exercise 1:** (3 Points)

Prove that the men-proposal algorithm is not DSIC for the right-hand side (the women). For this purpose, give an instance of the stable matching problem in which, by lying about her preferences during the execution of the men-proposing algorithm, a woman can end up with a man that she prefers over the man she would have ended up with had she told the truth.

**Exercise 2:** (4 Points)

Show that there are instances of the stable matching problem in which the Gale-Shapley Algorithm (men-proposing algorithm) runs for  $\Omega(n^2)$  iterations before terminating (with a stable matching). For this purpose, state an instance of the problem depending on  $n$  with suitable chosen preference orders and lower bound the number of iterations of the algorithm.

**Hint:** Consider an instance with  $|U| = |V| = n$ . Try to enforce exactly one rejection per iteration.

**Exercise 3:** (3 Points)

Recall the setting for Cake Cutting from Lecture 24. Show that if valuations are identical, i.e.  $v_i(\cdot) = v_j(\cdot)$  for all  $i, j \in N$ , then the notions of Proportionality, Envy-Freeness and Equitability coincide.

**Exercise 4:** (3 Points)

Consider the algorithm (which is also known as the *moving-knife algorithm*) given in Section 4 of Lecture 24 that determines a proportional allocation for any number of agents  $n$ .

Show that even in the case of three agents the allocation of the algorithm might not be envy-free.