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Algorithmic Game Theory

Summer Term 2024

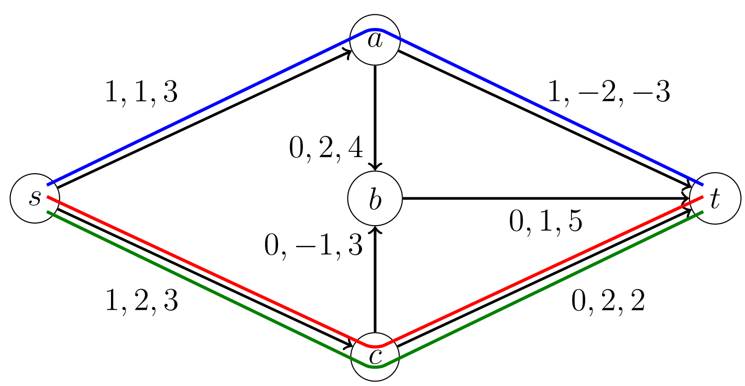
Tutorial Session - Week 0

You are supposed to work on these tasks in class together with your fellow students. Please find groups of 2 or 3 students!
If you do not know each other yet, each of you could start with a very quick introduction: What's your name? Do you study Computer Science or maybe something else (Maths, Economics,...)? Do you have any prior knowledge in Algorithmic Game Theory already or is this your first course in AGT?
Afterwards, you are supposed to discuss the exercises on this sheet. Note that you should see this also as a chance to talk about definitions, proof ideas and techniques used in the lecture in addition to only working out a formal solution for the tasks. If you do not know a definition or theorem by hard, feel free to open the lecture notes and have a look.

Exercise 1:
Consider the following symmetric network congestion game with players blue, red and green and their corresponding beginning strategies.

a) Formalize the network congestion game depicted below. For this purpose, specify the tuple $\Gamma = (\mathcal{N}, \mathcal{R}, (\Sigma_i)_{i\in\mathcal{N}}, (d_r)_{r\in\mathcal{R}})$. It suffices to state the delay function of a single resource/edge.

b) Find a pure Nash equilibrium by stating a sequence of best response improvement steps.



Exercise 2:
The following game is known as the Pollution Game. There are n players in this game and each player represents a country. For simplicity, we assume that each country has the following two choices: Either it agrees to set industry standards such that its pollution is controlled, or not. Agreeing to control pollution costs 3 for the country to set the industry standards. Each country that does not agree to control pollution adds 1 to the cost of all countries, including itself. Formally, every cost function $c_i : \Sigma \to \mathbb{Z}$ is given by

$$c_i(S) = \begin{cases} 3 + |\{j \in N|\text{j does not agree}\}| & \text{if i agrees,} \\ |\{j \in N|\text{j does not agree}\}| & \text{if i does not agree.} \end{cases}$$

Formulate this game as a congestion game and state a pure Nash equilibrium.