Thomas Kesselheim
Anna Heuser

April 20, 2026
Due date: April 27, 2026 at 8:00 p.m.

Algorithmic Game Theory

Summer Term 2026

Exercise Set 2

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner6.dfn.de/b/f27586e693d1c379bb7334799e4ff32a-1711809
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner6.dfn.de/b/4790db3a29e54531b388ea1db2d3d5ee-1711797

Exercise 1:
Consider the bimatrix game *Battle of the Sexes* given in Example 3.3 of the third lecture.

a) Compute a mixed Nash equilibrium by choosing probabilities for one player that will make the other player indifferent between his pure strategies (see Example 3.11).
b) Determine the probabilities of the mixed Nash equilibrium graphically by plotting the players' expected costs.

Exercise 2:
We define a strategy $s_i \in S_i$ of a normal-form cost-minimization game to be *strictly dominated*, if there exists a strategy s'_i such that $c_i(s'_i, s_{-i}) < c_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Prove that for all mixed Nash equilibria σ, there is no player $i \in N$ with a mixed strategy σ_i such that $\sigma_{i,s_i} > 0$ for a strictly dominated strategy $s_i \in S_i$.

Exercise 3:
Have a look at the proof of Nash's Theorem (4.3) in which normal-form payoff-maximization games are considered. Let $N = \{1, \ldots, n\}$ and $S_i = \{1, \ldots, m_i\}$ for all $i \in N$. The set of mixed states X can be considered as a subset of $\mathbb{R}^m$ with $m = \sum_{i=1}^n m_i$.

Show that X is convex and compact.

Exercise 4:
Consider the following game of $n \geq 2$ players. Every player selects, independently, a number from $\{1, ..., 1000\}$. The goal of each player is to have their number closest to the half of the average of all the selected numbers.
We define two variants of this game, depending on the tie-breaking rule. In the first rule, all players that are closest to the half of the average split evenly the payoff of 1. In the second tie-breaking rule, each player that is closest to the half of the average receives payoff of 1.

a) Determine all pure Nash equilibria of the game under the first tie-breaking rule.

b) Determine all pure Nash equilibria of the game under the second tie-breaking rule.

c) Are there other mixed Nash equilibria? Why? Why not?